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Determination of evaporation from a catchment water balance at a monthly time scale

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Abstract

A method is presented to determine total evaporation from the earth's surface at a spatial scale that is adequate for linkage with climate models. The method is based on the water balance of catchments, combined with a calibrated autoregressive rainfall–runoff model. The time scale used is in the order of decades (10 days) to months. The rainfall–runoff model makes a distinction between immediate processes (interception and short term storage) and the remaining longer-term processes. Besides the calibrated rainfall–runoff model and the time series of observed rainfall and runoff, the method requires a relation between transpiration and soil moisture storage. The method is applied to data of the Bani catchment in Mali, a sub-catchment of the Niger river basin.

Introduction

The determination of spatially integrated evaporation is an important prerequisite for the correct simulation of atmospheric processes in climate models. Evaporation supplies the necessary feedback of moisture to the atmosphere to sustain rainfall (e.g. Salati et al., 1983; Lettau et al., 1979; Brubaker et al., 1993; Savenije & Hall, 1994). Without evaporation, rainfall would rapidly decline inland, resulting in higher runoff in the coastal zone and drastic reductions of continental rainfall. On the west African continent, if evaporation were completely excluded, rainfall would not reach much further than several hundreds of kilometres and the Sahara desert would cover all countries in the Sahelian belt (Savenije, 1995,1996). In climate and weather models, the inclusion of moisture feedback through evaporation is recognised as a necessity to simulate continental rainfall adequately (e.g. Beljaars and Holtslag, 1991). At present, the correct representation of moisture feedback into climate and weather models is one of the major challenges for both meteorologists and hydrologists.

There are broadly three approaches to the establishment of evaporation time series at these temporal scales:

1. By spatial upscaling of point observations (micro-scale). There are several ways of determining actual evaporation from research sites, either through direct measurement or indirectly by monitoring the components of the energy balance equation. Although accurate time series can be obtained for the site under consideration, the main problem remains the spatial upscaling.
2. By monitoring and modelling the components of the energy balance equation (at meso- or macro-scale) on the basis of remote sensing information, combined with ground-truth (e.g. Bastiaanssen, 1995; Peters, 1995). Although science-based, these methods have to make use of semi-empirical relations and incorporate a considerable amount of uncertainty because of lack of ground-truth.
3. By establishing the different components of the water balance equation at the spatial scale of river basins (meso-scale).

The solution to the problem probably lies in a combination of these three methods. The remote sensing approach has the right scale for input into climate models, but it contains many semi-empirical components that require verification and calibration by evaporation measurements at the appropriate scale. The river basin approach has the most adequate scale for this ground-truth, but it requires, in its turn, verification from plot-size observations and conversion to the grid-size of the climate model.

In the following, a method for the simulation of actual evaporation from a river basin, or sub-basin is presented which is adequate for this purpose, and which moreover can be used to analyze the effect of land use on changes in evaporation and runoff.

Theory

The water balance of a catchment is defined by:

$$\frac{\mathrm{d}S}{\mathrm{d}t} = P - E - Q \tag{1}$$

where:

S: the total sub-surface storage of moisture in the catchment $S = S_u + S_g$
S_u: the available moisture for transpiration in the root zone (exceeding the wilting point) (mm)
S_g: the remaining subsurface storage $S_g = S - S_u$ (mm)
P: the areal average rainfall (mm s^{-1})
E: the areal average evaporation (mm s^{-1})
Q: the average runoff from the catchment (including groundwater seepage) (mm s^{-1})
t: time (s)

In S, for this case, the surface water storage is not considered, since in catchments without major water bodies, surface water storage (in ponds etc.), at the timescale considered, is transferred to direct (open water) evaporation and interception, which is included in E. If, however, there are major open water bodies in the catchment, then S can be expanded to contain a component S_s, which can generally be assessed easily by a reservoir balance equation. In addition, snow accumulation is not considered in this approach.

The water balance equation can be elaborated, distinguishing between groundwater storage and storage in the unsaturated zone, and by separating the evaporation into interception I and transpiration T:

$$\frac{\mathrm{d}\,S_u}{\mathrm{d}\,t} = P - I - T - \left(Q + \frac{\mathrm{d}\,S_g}{\mathrm{d}\,t}\right) \tag{2}$$

In gauged catchments, time series of P and Q may exist at the required spatial and temporal scales. The variation of the storage and the evaporation, however, is much more problematical. What can be derived easily is the evaporation at the timescale of a hydrological year, since then storage fluctuations may be neglected. However, the problem remains as to how to disaggregate the annual evaporation to shorter time steps.

Rainfall–runoff modelling may present a solution, in that it can supply estimates of I and $\mathrm{d}S_g/\mathrm{d}t$. If subsequently a relation can be established between T and S_u, then the equation can be solved.

Rainfall–runoff modelling

The RAINRU model on spreadsheet (Savenije, 1995) presents a simple but adequate tool to determine these parameters. In the following, the theory of the approach is presented for a monthly time step.

Since the runoff in a particular month depends not only on the rainfall for that month, but (through storage) also on the rainfall for the preceding months, a linear transfer function is used:

$$Q(t) = \sum_{i=0}^{n} b_i \times \mathrm{Max}\big(P(t-i) - D, 0\big) \tag{3}$$

where $i \in [0,1,2,\ldots n]$ is the counter of backward time steps from the start of the rainfall at time step t. The parameters Q, P, and D are expressed in mm month^{-1}. The coefficients b_i are determined through multiple stepwise regression. In fact, the regression coefficients b_i are hydrograph coefficients resulting from a net rainfall: $N = \mathrm{Max}(P{-}D, 0)$ (see Appendix A). Some hydrologists use net rainfall for the amount of rainfall that actually produces the runoff; here net rainfall is defined as the part of the rainfall that enters into the rainfall–runoff process after the immediate process of moisture recycling to the atmosphere within the timestep dt has been subtracted. The net rainfall N is the rainfall which exceeds a threshold value D, which consists of interception and other immediate feedback to the atmosphere during the time step considered. If $P{-}D<0$, than the net rainfall is zero. Consequently, D is the threshold loss on a monthly basis (the maximum amount of immediate moisture feedback per month), b_i is the coefficient that determines the contribution of the net rainfall in month $t{-}i$ to the runoff in month t (the combined effect of surface runoff and groundwater seepage). In fact, D is the separation between the immediate and longer-term processes for the time-scale considered. The immediate processes consist of the interception, evaporation from temporary surface storage (pools), bare soil evaporation and immediate transpiration (within the timescale considered) from the surface layer.

For reasons of simplicity, in this paper, D is taken as a constant in time. This is, however, not necessary. One could, to account for seasonal fluctuations (for example to account for leaf area indexes or the number of rain days per month), establish fixed values of D for different periods in the year.

Eq. (3) can be simplified to:

$$Q(t) = \sum_{i=0}^{n} b_i \times N(t-i) \tag{4}$$

The net runoff coefficient c, on a water year basis, is defined as:

$$c = \sum_{t=1}^{m} Q(t) \Big/ \sum_{t=1}^{m} N(t) \tag{5}$$

being the proportion of accumulated catchment discharge to accumulated net rainfall. The summation is done over m monthly values (m is a multiple of 12), comprising at least one hydrological year. The net runoff coefficient differs from the 'traditional' runoff coefficient in that it is based purely on the net rainfall, the rainfall which remains after the immediate moisture feedback processes have been subtracted.

In Appendix A it is demonstrated mathematically that:

$$c = \sum_{i=0}^{n} b_i \qquad (6)$$

By reasoning, one can reach the same conclusion considering that the proportion of the accumulated discharge to the accumulated net rainfall is equal to the amount of discharge that a unit amount of net rainfall generates, which, according to unit hydrograph theory, equals the sum of the hydrograph coefficients. Obviously c should not be larger than unity.

The coefficients b_i are determined through multiple linear regression. The threshold D is determined iteratively until maximum explained variance is obtained. As an example, the result of rainfall–runoff analysis, on a monthly time-step basis, is presented in Table 1 for the Bani catchment.

The Bani catchment lies in West Africa, mostly within Mali. The Bani, a tributary to the Niger, has its confluence with the Niger at Mopti. The part of the Bani modelled here is the area upstream of Douna (near Ségou) just before it enters into the Niger floodplain. The catchment area amounts to 101 600 Mm^2 and is covered mostly with savanna vegetation.

Table 1 Regression results for the Bani Catchment

Regression Output:					
Threshold rainfall	90mm				
Standard error of estimate	6.6				
R^2	0.94				
No. of observations	120				
Degrees of freedom	115				
	b_0	b_1	b_2	b_3	b_4
Coefficients	0.00	0.16	0.13	0.03	0.02
Standard error of estimate	0.01	0.01	0.01	0.01	0.01

The average rainfall over the catchment, for the period considered of 1952–1961, amounted to 1311 mm/a; the average runoff over the same period was 237 mm/a, hence the average evaporation was 1074 mm/a, and the runoff coefficient 0.18.

Fig. 1a shows the simulated and the measured hydrographs for the Bani catchment. The values of D and c in the Bani catchment are 90 mm and 0.32 respectively, meaning that the monthly capacity of immediate rainfall feedback is 90 mm and that 32% of the remaining water ends up, eventually, in the river at Douna. The memory of the system is four months, meaning that the effect of rainfall on river discharge is felt in the river at Douna until four months after the end of the rain.

Critical hydrologists may not be impressed by the fit of measured and computed runoff in Fig. 1a. Reasons for the imperfect fit are several, a very deficient rainfall gauging network, based on only three stations and the imperfect rating and water level recording at the control section. However, it is not the intention of this paper to show that the rainfall–runoff model of equation (3) is adequate; it is a linear transfer function of which the applicability to rainfall–runoff processes does not need to be proven (although Fig 1b shows a split record validation for the Bani river with the coefficients of D = 90 mm and c = 0.32 applied to the rainfall of 1962–1971). Rather, the intention is to show that the use of this simple linear transfer model can isolate immediate moisture feedback from overall evaporation and disaggregate annual total evaporation over a catchment into monthly or decade evaporation, which is done in the following paragraphs.

Establishment of interception and groundwater storage

It follows from the above theory that the immediate moisture feedback to the atmosphere (interception, which on a monthly time scale includes evaporation from pools and bare soil evaporation) during a timestep dt can be described by:

$$I = \mathrm{Min}(P,D) \qquad (7)$$

meaning that the interception is equal to the rainfall if it is less than the threshold value and equal to D if it is more. It also follows that the amount which runs-off superficially or recharges the groundwater as a result of the rainfall in a time-step dt is equal to the product of the net rainfall and the net runoff coefficient, being the part of the net rainfall that contributes to the runoff process:

$$Q + \frac{dS_g}{dt} = c \times \mathrm{Max}(P - D, 0) \qquad (8)$$

Since c and D have been determined through regression, these components of the water balance in equation (2) can be determined.

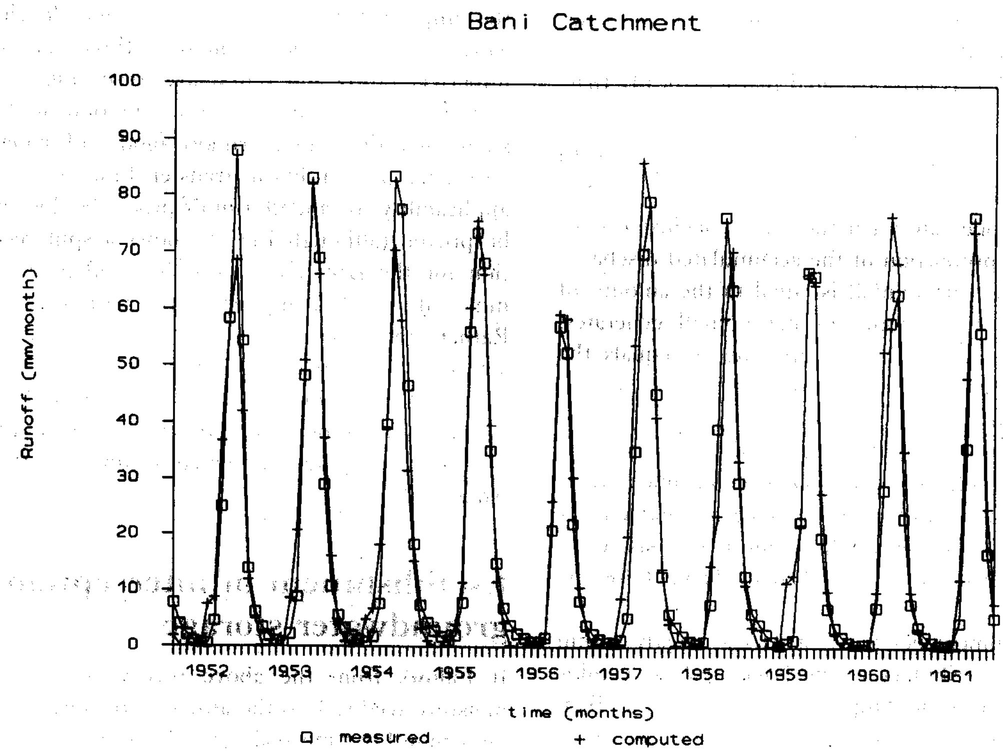
Transpiration as a function of S_u

If there is no shortage of water in the unsaturated zone, transpiration can be potential T_p. The potential transpiration is the difference between the potential evaporation E_p derived from the energy balance and the actual interception:

$$T_p = E_p - I \qquad (9)$$

If, however, shortage through moisture depletion occurs, the relative transpiration (T/T_p) reduces as well. If a

(a)



(b)

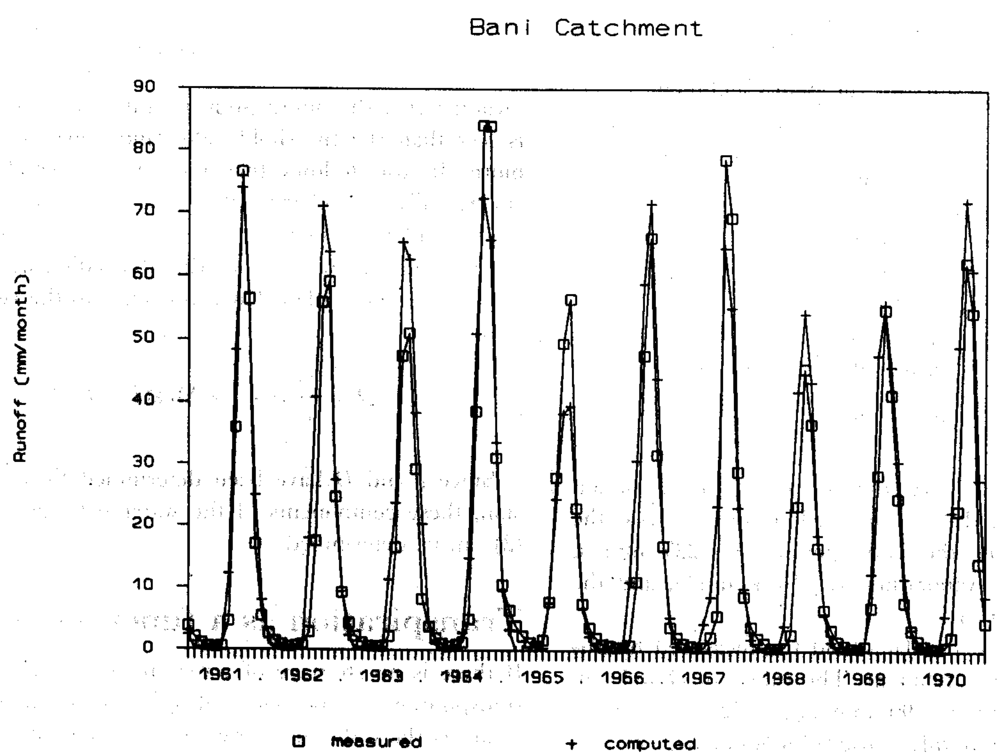


Fig. 1 (*a*) *Simulated and Measured discharge in the Bani river at Douna* (*b*) *Split record validation of Bani river at Douna for the period 1961–70 on the basis of data of 1952–61* linear relation between storage and relative transpiration is assumed (Rijtema & Aboukhaled, 1975), the following relation is arrived at (see Fig. 2):

$$T = \mathrm{Min}(a \times T_p \times S_u, T_p) \quad (10)$$

This linear proportionality is generally used by irrigation engineers (Doorenbos and Kassam, 1979). In the theory of Rijtema & Aboukhaled, the coefficient a corresponds to:

$$a = \frac{1}{(1-p)S_m} \quad (11)$$

where S_m is the maximum available soil moisture (corresponding to the difference between wilting point and field capacity) and p is the amount of the maximum available moisture that is readily available for transpiration. As a reasonable assumption $p \approx 0.5$. The parameter a has the dimension $(\mathrm{mm})^{-1}$.

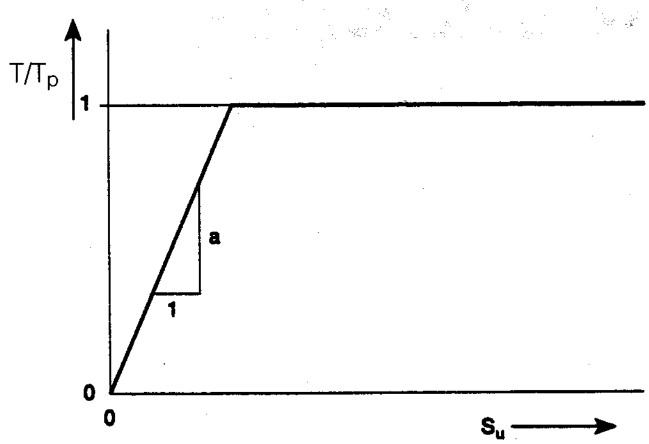


Fig. 2 *Relation between transpiration T and soil moisture storage* S_u

Assuming E_p can be determined as a function of time (for instance through remote sensing as by Peters, 1995 or Bastiaanssen, 1995, or on the basis of pan evaporation), the only calibration coefficient that remains to be determined is the proportionality constant a. Initially, indications of reasonable values for a should be obtained from equation (11). Subsequently, a should be determined through calibration of the model. Finally, the values obtained should be verified by field observations.

Solution of the water balance equation

The water balance equation (2) is solved stepwise with a timestep dt equal to 1 (1 month or 1 decade (10 days)):

$$\begin{aligned} S_u(t) = {} & S_u(t-1) + P - \mathrm{Min}(D,P) \\ & - \mathrm{Min}(a \times T_p \times S_u(t-1), T_p) - c \\ & \times \mathrm{Max}(P - D, 0) \end{aligned} \quad (12)$$

Note that for the determination of the actual transpiration, the storage in the unsaturated zone of the previous time step is used.

Since it can be shown that

$$P - \mathrm{Min}(D,P) = \mathrm{Max}(P - D, 0) \quad (13)$$

equation (12) can be simplified into:

$$\begin{aligned} S_u(t) = {} & S_u(t-1) - \mathrm{Min}(a \times S_u(t-1), T_p) + \\ & (1-c) \times \mathrm{Max}(P - D, 0) \end{aligned} \quad (14)$$

which has only one calibration coefficient a. Besides S_u, the other variables are known either from observations or from calibration of the RAINRU model. Assuming a starting value of S_u at $t = 0$, equation (14) can be solved for subsequent time steps. Subsequently, the transpiration and the total evaporation for the whole time series can be computed.

Fig. 3 shows the variation over time of the observed areal average rainfall, observed runoff and total computed evaporation for the Bani catchment. The potential evaporation E_p has been determined with the CROPWAT model based on the method of Penman-Monteith. The value of a used in the simulation is 0.004. This value is obtained by substitution of $p = 0.5$ and $S_m = 500$ mm in equation (11). This latter value is in agreement with the soil moisture fluctuation in the root zone of Fig. 4, which presents both the variation of the total catchment evaporation and the soil moisture storage in the root zone.

Conclusion

In fact what the method does is to use a statistical model, equation (3), to provide estimates of deterministic parameters, I and $Q + \mathrm{d}S_g/\mathrm{d}t$, for use in the water balance equation (2). The combined use of the linear transfer model and the water balance thus provides the basis to disaggregate the annual avaporation into monthly or decade values. The remaining problem of how to link soil moisture availability to transpiration is solved by using a relation such as eq. (10). The water balance equation then yields a time series of evaporation and soil moisture storage at a catchment scale at a monthly or decade time step.

The question one may ask is whether the statistical estimates of I and $Q + \mathrm{d}S_g/\mathrm{d}t$ are sufficiently accurate to allow disaggregation of E at the time step considered. Confrontation with other methods of evaporation determination on the basis of remote sensing would be an interesting next step by which both methods could be improved. Since remote sensing techniques can only be applied for the time at which an image is taken and provided there is no cloud cover, the method presented here could also be extremely useful for the temporal integration of areal evaporation determined through remote sensing. Finally, the method can be quite useful to

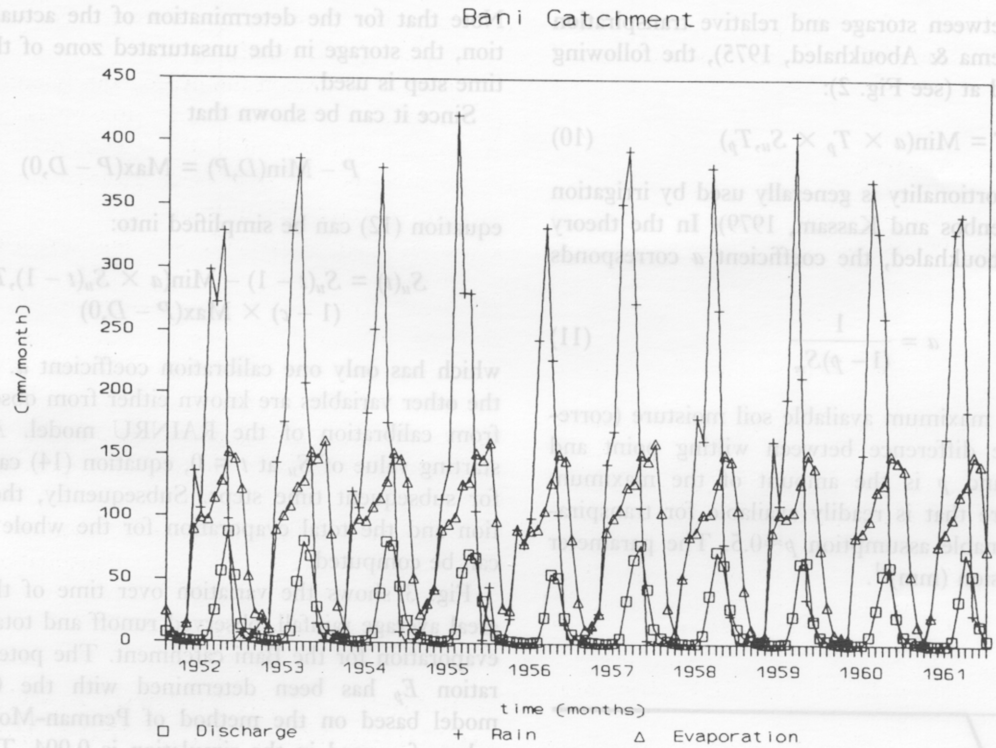


Fig. 3 *Simulation of Rainfall Evaporation and Runoff in the Bani Catchment at Douna*

Fig. 4 *Simulation of Evaporation and Soil Moisture Storage in the Bani Catchment*

combine with climate models, which have not yet solved the feedback of moisture from the earth's surface to full satisfaction.

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Notation

- a proportionality factor (mm^{-1})
- b_i regression coefficient, hydrograph coefficient
- c net runoff coefficient (Q/N)
- D threshold rainfall, maximum amount of immediate moisture feedback, or potential interception (mm/month)
- E areal average total evaporation ($I + T$) (mm/month)
- E_p areal average potential evaporation (mm/month)
- I areal average immediate moisture feedback, or interception (mm/month)
- i counter of time in the hydrograph (month)
- m number of time steps in the time series (month)
- N net amount of rainfall ($P - I$) (mm/month)
- n number of time steps in the hydrograph (month)
- P areal average rainfall (mm/month)
- p proportion of available soil moisture that is readily available
- Q areal average runoff (mm/month)
- S total sub-surface catchment storage (mm)
- S_g subsurface storage in the catchment below the root zone $S_g=S-S_u$ (mm)
- S_m maximum available soil moisture in the root zone
- S_u moisture storage in the root zone above wilting point (mm)
- T areal average transpiration (mm/month)
- T_p potential areal average transpiration ($E_p - I$) (mm/month)
- t time counter (month)

Appendix A: Net runoff coefficient equalling the sum of the regression coefficients

When equation (3) is accumulated over m months, completing an entire number of hydrological years (m is a multiple of 12), this equation modifies into:

$$\sum_{t=1}^{m} Q(t) - \sum_{t=1}^{m}\sum_{i=0}^{n} b_i \times N(i,t) \tag{A.1}$$

where:

$$N(i,t) = \mathrm{Max}(P(t-i) - D, 0) \tag{A.2}$$

It follows from this definition that:

$$N(i,t) = N(i+1, t+1) \tag{A.3}$$

In addition:

$$N(0,t) = N(t) = \mathrm{Max}(P(t) - D, 0) \tag{A.4}$$

Elaboration of equation (A.1) yields:

$$\begin{aligned}\sum_{t=1}^{m} Q(t) = {} & b_0N(0,1) + b_1N(1,1) + b_2N(2,1) + \ldots + b_nN(n,1) + \\ & b_0N(0,2) + b_1N(1,2) + b_2N(2,2) + \ldots + b_nN(n,2) + \\ & b_0N(0,3) + b_1N(1,3) + b_2N(2,3) + \ldots + b_nN(n,3) + \\ & \vdots \\ & b_0N(0,m) + b_1N(1,m) + b_2N(2,m) + \ldots + b_nN(n,m)\end{aligned} \tag{A.5}$$

Rearrangement of equation (A.5) leads to:

$$\sum_{t=1}^{m} Q(t) = b_0 N(0,1) + b_1 N(1,2) + b_2 N(2,3) + \ldots + b_n N(n,1+n) +$$
$$b_0 N(0,2) + b_1 N(1,3) + b_2 N(2,4) + \ldots + b_n N(n,2+n) +$$
$$b_0 N(0,3) + b_1 N(1,4) + b_2 N(2,5) + \ldots + b_n N(n,3+n) +$$
$$\vdots$$
$$b_0 N(0,m) + b_1 N(1,m+1) + \ldots + b_n N(n,m+n) \qquad \text{(A.6)}$$

In this rearrangement, some values of the last months of the time series (not more than n values covering the length of the hydrograph n) are exchanged with the corresponding values in the last months of the hydrological year preceding the time series: $N(i,t+m) = N(i,t)$ where $1 < t < n$. This is acceptable since in the months preceding the start of the hydrological year, the net rainfall is negligible. Moreover if the time series consists of more than one hydrological year, the influence, although negligible in itself, will be reduced even further.

Substitution of equation (A.3) yields:

$$\sum_{t=1}^{m} Q(t) = b_0 N(0,1) + b_1 N(0,1) + b_2 N(0,1) + \ldots + b_n N(0,1) +$$
$$b_0 N(0,2) + b_1 N(0,2) + b_2 N(0,2) + \ldots + b_n N(0,2) +$$
$$b_0 N(0,3) + b_1 N(0,3) + b_2 N(0,3) + \ldots + b_n N(0,3) +$$
$$\vdots$$
$$b_0 N(0,m) + b_1 N(0,m) + b_2 N(0,m) + \ldots + b_n N(0,m) \qquad \text{(A.7)}$$

Finally, substitution of equation (A.4) yields:

$$\sum_{t=1}^{m} Q(t) = \sum_{t=1}^{m} \sum_{i=0}^{n} b_i \times N(t) = \sum_{i=0}^{n} b_i \sum_{t=1}^{m} N(t) \qquad \text{(A.8)}$$

And hence it follows from equation (5) that:

$$c = \sum_{i=0}^{n} b_i \qquad \text{(A.9)}$$