



# Comment on the application of the Szilagyi–Jozsa advection–aridity model for estimating actual terrestrial evapotranspiration in ‘Estimating actual, potential, reference crop and pan evaporation using standard meteorological data: a pragmatic synthesis’ by McMahon et al. (2013)

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**Abstract.** In the paper by McMahon et al. (2013, supplementary sections S8 and S19, worked example 8), the Szilagyi–Jozsa advection–aridity model (Szilagyi, 2007; Szilagyi and Jozsa, 2008) was not applied in the worked example as intended by author J. Szilagyi. This commentary seeks to clarify the issue and provide the correct procedure.

## 1 Background

In the paper by McMahon et al. (2013, supplementary sections S8 and S19, worked example 8), the Szilagyi–Jozsa (SJ) advection–aridity model (Szilagyi, 2007; Szilagyi and Jozsa, 2008), which is a modification of the original advection–aridity model of Brutsaert and Stricker (1979), was not applied in the worked example as intended by author J. Szilagyi. This commentary seeks to clarify the issue and provide the correct procedure.

The SJ model is based on the complementary relationship (Bouchet, 1963; Szilagyi, 2007) as follows:

$$ET_{Act}^{SJ} = 2 E_{PT}(T_e) - E_{Pen}, \quad (1)$$

where  $ET_{Act}^{SJ}$  is actual evapotranspiration ( $\text{mm day}^{-1}$ ),  $E_{PT}(T_e)$  is wet environment evaporation ( $\text{mm day}^{-1}$ )

estimated by the Priestley–Taylor (PT) method at the equilibrium temperature, or wet environment surface temperature,  $T_e$  ( $^{\circ}\text{C}$ ) (Priestley and Taylor, 1972), and  $E_{Pen}$  is potential evapotranspiration ( $\text{mm day}^{-1}$ ) estimated by the Penman method using the 1948 wind function (Penman, 1948). Equations to compute  $E_{PT}(T_e)$  and  $E_{Pen}$  are presented in McMahon et al. (2013, Eqs. 4 and 6, respectively) with details regarding Penman's (1948) wind function given in Supplementary section S4. The equilibrium temperature is the temperature of the evaporating surface at which the net rate of heat exchange (by shortwave and longwave radiation, conduction and evaporation) is zero (Edinger et al., 1968). According to Sweers (1976, p. 377), equilibrium temperature is never achieved because daily fluctuations in meteorological conditions disrupt the formation of equilibrium conditions.

To evaluate  $T_e$ , Szilagyi and Jozsa (2008) utilised the Bowen ratio (Bowen, 1926) for a small lake or sunken pan and found that the equilibrium surface temperature,  $T_e$ , could be estimated iteratively on a daily basis from Szilagyi and Jozsa (2008, Eq. 8):

$$\frac{R_n}{\lambda E_{Pen}} = 1 + \frac{\gamma (T_e - T_a)}{v_e^* - v_a}, \quad (2)$$

where  $R_n$  is the available energy ( $\text{MJ m}^{-2} \text{ day}^{-1}$ ),  $E_{Pen}$  is the Penman evaporation ( $\text{mm day}^{-1}$ ) based on  $T_a$ ,  $T_e$  and  $T_a$

**Table 1.** Comparison of the Szilagyi–Jozsa (SJ) advection–aridity actual evapotranspiration model incorporating  $T_e \leq T_a$  with uncorrected SJ and with the Brutsaert–Stricker (BS) advection–aridity model.

Meteorological station (1)	Mean annual precipitation (mm $y^{-1}$ ) (2)	BS (3)	SJ (uncorrected) (4)	SJ ( $T_e \leq T_a$ ) (5)
9021 Perth Airport	731	736	701	655
14015 Darwin Airport	1777	1515	1531	1471
15590 Alice Springs Airport	279	537	360	321
40842 Brisbane Aero	839	1070	1107	1053
86282 Melbourne Airport	512	349	328	292
94069 Grove	696	630	704	627

are respectively the equilibrium and air temperatures ( $^{\circ}\text{C}$ ),  $v_e^*$  is the saturation vapour pressure (kPa) at  $T_e$ ,  $v_a$  is the actual vapour pressure (kPa) at  $T_a$ ,  $\lambda$  is the latent heat of vaporization ( $\text{MJ kg}^{-1}$ ), and  $\gamma$  is the psychrometric constant (kPa  $^{\circ}\text{C}^{-1}$ ). The correct procedure should ensure that  $T_e$  is capped at  $T_a$ .

## 2 Basis of $T_e \leq T_a$

The advection–aridity (AA) model of Brutsaert and Stricker (1979) assumes that the available energy at the evaporating surface, which is used for sensible and latent heat fluxes (soil conduction is assumed to be negligible), stays quasi-constant as the environment dries following an initial wet condition under minimal horizontal energy advection. By extending the quasi-constant net surface radiation term over a patch of land that retains its original moisture status as the environment dries around it, one can expect that the surface temperature ( $T_e$ ) of the wet patch will remain constant during the drying out of the surrounding land.

In the AA model the evapotranspiration rate of the wet patch with a regional extent is given by the Priestley–Taylor method. However, the coefficient in the PT equation was derived under wet environment conditions, yet in the AA model PT is utilised under non-wet conditions, with the actual air temperature ( $T_a$ ) being higher than it would be under wet conditions. This affects the slope of the saturation vapour pressure curve,  $\Delta$ . Szilagyi and Jozsa (2008) and Szilagyi et al. (2009) suggested that the unknown wet environment surface temperature ( $T_e$ ) be back-calculated, assuming it to be time-invariant under a constant net surface energy term.  $T_e$  is used to evaluate the  $\Delta$  term in the PT equation, as a proxy for the unknown wet environment air temperature. It should be noted that such a correction is not necessary in the Penman equation, because it was derived for wet surfaces under typically non-wet environmental conditions with air temperature, moisture and radiation measurements upwind of the wet surface.

Since the environmental conditions do not always satisfy the assumptions in the method (e.g. horizontal energy advection may be significant) and the measurements are not perfect, it can happen that the back-calculated  $T_e$  becomes higher than the actual air temperature,  $T_a$ . In this situation it is necessary to restrict the value of  $T_e$  to being equal to or less than  $T_a$ , as was done implicitly by Szilagyi and Jozsa (2008) and Szilagyi et al. (2009), but more explicitly in Huntington et al. (2011). This restriction on  $T_e$  is required to ensure that the PT equation estimates wet environment evaporation. If  $T_e$  were allowed to exceed  $T_a$ , the PT equation would not be representative of wet environment conditions. As a result, for the same albedo values, the modified AA model (SJ model) can never yield larger ET rates than the original AA model of Brutsaert and Stricker (1979) if the same PT alpha value is adopted in each model.

## 3 Application of the corrected Szilagyi–Jozsa model

To illustrate the importance of the  $T_e \leq T_a$  constraint in the SJ model, we applied the constraint to the data for the six meteorological stations analysed in McMahon et al. (2013, Table S13 in the Supplement). The mean annual actual ET estimates based on the corrected SJ model (column 5) are compared with uncorrected estimates (column 4) and the BS estimates (column 3) in Table 1 where the values in columns 3 and 4 are reproduced from Table S13 in the Supplement of McMahon et al. (2013). In this comparison it should be noted that for the BS model a PT alpha value of 1.28 as recommended by Brutsaert and Stricker (1979) was adopted and for the SJ model an alpha of 1.29 (as the complementary relationship of Eq. (1) was applied at the daily time step) (Szilagyi and Jozsa, 2008, p. 185).

As expected, the mean annual actual ET estimates for the corrected SJ model are less than those for the BS model, although for some days the SJ estimates were greater than BS values (results not shown in the table). This is the result of the two models using different alpha values in the Priestly–Taylor equation. As expected, when the same alpha values were adopted in the two models, ET estimates for SJ

were always less than those for BS. Furthermore, four of the corrected values of SJ (Table 1, column 5) are less than the mean annual precipitation, which indicates that the model is physically plausible.

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