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Using interactive recession curve analysis to specify a general catchment storage model

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Abstract

An analysis of hydrograph recessions can be used to identify the parameters of a conceptual catchment storage model and, with the advent of large-scale digital data storage and automated logging systems, it has become desirable to automate recession curve analysis. Various studies have thus reported algorithms used to infer 'baseflow' storage models automatically from recession data. Such algorithms commonly operate by maximising the fit of measured recession data to some *a priori* function. Here, an alternative approach is taken in which the appropriate *form* for a catchment saturated zone store is investigated by combining observed recession data to form a Master Recession Curve (MRC). This is done within a software package that offers automated functions to help select recession periods suitable for inclusion within the MRC. These recession periods are combined automatically to form a 'prototype' MRC, which can be modified interactively to overcome problems such as unrepresentative or sparse data. The master recession for a catchment is used to calculate an empirical catchment–averaged discharge–relative storage ($Q\Delta S$) relationship. The method is considered to be general because the $Q\Delta S$ relationship may be of arbitrary form. Examples are given, showing the derivation for three catchments of different $Q\Delta S$ functions.

Introduction

There is a long history of attempts to interpret stream hydrograph recessions in order to obtain characteristic functions for catchments. Comprehensive reviews have been given by Tallaksen (1995) and Hall (1968). Recession curve analysis is difficult because of the great variability that can exist between individual recession periods. This variability makes it hard to infer, physically or conceptually, meaningful underlying functions from recession data. Despite the difficulties, methods have been proposed for the quantification of recession curves. Tallaksen (1995, p.349) states that:

> 'The quantification of the recession curve involves the selection of an analytical expression, derivation of a characteristic recession and optimisation of the recession parameters.'

This suggests one approach to the problem in which a characteristic recession, which is based on discharge observations, can be interpreted in terms of an underlying analytical function, selected *a priori*. This underlying function may be a conceptual storage model or be derived from physical principles, although certain forms of conceptual store can be shown to be equivalent to more physical descriptions of groundwater flow, (e.g. Sloan and Moore, 1984). Characterisation of an *a priori* analytical function is the approach adopted in some studies of automated recession analysis (Nathan and McMahon, 1990, Institute of Hydrology, 1980) and has the obvious advantage that observed recession periods may be compared to the theoretically assumed function and their suitability for that model evaluated. Any periods of recession flow that do not conform to the underlying analytical function are rejected in this approach.

An alternative approach is to use recession data to help in defining the form of an unknown underlying function, rather than seek to fit these data to an *a priori* function. This is a valid research aim since there is evidence that the underlying function may, in some cases, be sufficiently complex to give rise to 'compound' recession curves as found by Trainer and Watkins (1974) and suggested by Brutsaert and Nieber (1977). Such compound recession functions were used to derive predictive 'Inflow-Storage-Outflow' relationships by Lambert (1972).

It will be assumed that the underlying function to be identified is a lumped store. Such a store has the advantage that it is conceptually simple, but may have an arbitrarily complex form if expressed as a look-up table. By interpreting discharge from this store to represent saturated zone drainage, it is possible to define certain physical conditions, to help select recession data suitable for analysis (see below).

This paper describes an interactive software package used to derive a catchment saturated zone store from recession curve data. The package allows large amounts of recession data to be managed efficiently and helps the operator to select suitable data using automated filters. Selected recessions can be combined automatically to form a prototype master curve, adopting the principles of the well-established matching strip method (Snyder, 1939, Toebes and Strang, 1964). It is left to the operator to improve the MRC on the basis of their expertise and judgement by using a set of interactive functions.

The subjective element of the analysis is a consequence of the inherent difficulties in analysing recession curves, but the aim of the proposed system is to allow the exploration of potentially large data sets in a convenient way. Where there are sufficient data, the system can be applied quickly to give reproducible results

Saturated zone storage concepts

Certain conceptual storage functions correspond to more physical descriptions of saturated flow. For example, a linear store gives rise to recession flows that are exponential in time when solved with a continuity equation under zero recharge, whilst the same result can be obtained as an exact solution of the linearised Dupuit-Boussinesq groundwater flow equations (Brutsaert and Nieber, 1977).

Perhaps the most general storage function commonly encountered has the form $Q = S^N/T$ where the discharge Q is a function of storage S, a parameter N and T, which is the time constant of the store. This function can be solved exactly for recession flows over time. However, an even more general conceptual store can be specified in terms of a discharge–relative storage ($Q\Delta S$) relationship which may be of arbitrary form. Further, $Q\Delta S$ relationships that conform to analytical functions can be distinguished from those that do not; the latter type may be approximated by purely empirical functions or expressed in the form of look-up tables. Lambert (1969, 1970) specifically aimed to derive the parameters of a catchment scale storage model from recession analysis but used a tabular method for predicting flows that might be adapted for use with a general $Q\Delta S$ relationship.

The Master Recession Curve

To infer a saturated zone $Q\Delta S$ relationship from recession curve data, it is necessary that these data fulfil certain conditions. If the $Q\Delta S$ relationship is to be held to represent the subsurface drainage system, then the flow data used in its identification should ideally be free from external influences. The most significant influences are evaporation and storm effects, such as rainfall or snowmelt-induced surface or near surface flows.

It is unlikely that a single recession curve can ever fulfil these conditions over more than a relatively limited range of flows, especially in catchments where there are frequent storms. A method is therefore required to combine individual recession periods (not necessarily entire recessions) to synthesise a master recession curve over a wider range of flows.

A suitable computerised method may be based on the following procedures:

1. Recession periods are extracted from flow data, along with corresponding measurements of rainfall (or snowmelt), evaporation (or a proxy) and any other useful information such as indications of data quality, gauge underflow etc.
2. Recession curves that are unsuitable for analysis are rejected. This process may be automated using filters that reject curves on the basis of some objective criterion. It will also be desirable to allow recession data to be examined in detail by the hydrologist who may be able to identify problems in the data that might not be found by a fully-automated algorithm.
3. Recessions have to be combined to form a master curve. Automated procedures may be helpful here, although the operator should be able to intervene so as to exercise judgement in the final analysis.

There are two approaches to the formation of the master curve (stage 3 above). In one approach, the properties of an ensemble of curves are quantified in some way, for example by statistical analysis (although it may be difficult to interpret the statistics of a set of curves without assuming a prior recession model). An alternative approach is the matching strip technique, traditionally carried out using transparent overlays, in which individual recessions are aligned to form a continuous master curve.

The matching strip technique can be criticised because it is inherently subjective. However, some subjectivity is inevitable in an investigation of recession data with a minimum of prior assumptions. Within a graphical 'expert system' framework, the subjectivity of the method is arguably an advantage since an experienced operator can look at any recession curve in detail and assess the suitability and quality of the data. The matching strip method preserves continuous periods of the measured data, which can help the operator in this assessment. This facility may be especially useful where the available data are sparse or of poor quality.

Implementation in the 'MRCtool' system

Stages of 1–3 of the recession analysis procedure have been implemented in a package called the 'MRCtool' system. 'MRCtool' is coded within the MATLAB package and can be run on any platform supporting MATLAB. The system, based on three display windows containing mouse-activated functions, is shown in outline in Figure 1 and its main functions are described below.

RECESSION CURVE IDENTIFICATION AND SELECTION

The data required by MRCtool are, as in stage 1 above, concurrent time series of discharge, rainfall (or snowmelt) and evaporation (or some proxy), measured at regular intervals. Rainfall and potential evapotranspiration (PET) will be assumed for this paper. These data are first scanned to find recessions, which are defined simply as any period of decreasing measured flow. Recession periods are then plotted on the screen together with coincident rainfall and PET data.

Recession curves that are associated with periods of high potential evaporation or that are subject to sustained rainfall should be rejected from the MRC data set. The use of an interactive graphical display in the MRCtool system allows the recession curve selection procedure to be conveniently semi-automated.

A pair of slider controls is set up in the first MRCtool window to allow the data to be filtered in three ways. Firstly, an exclusion criterion is set to reject curves on the basis of the PET associated with each recession. This criterion is presently a maximum permitted value for the volumetric ratio of evaporation to discharge, summed over the duration of each recession. A second slider acts in the same way to filter data associated with high rainfall volumes. Thirdly, a filter is provided to specify the minimum length of recession curves to be included in the MRC. This filter helps identify sustained recession periods which may be preferable, since their use minimises the number of intervention points contained within the final, synthetic MRC.

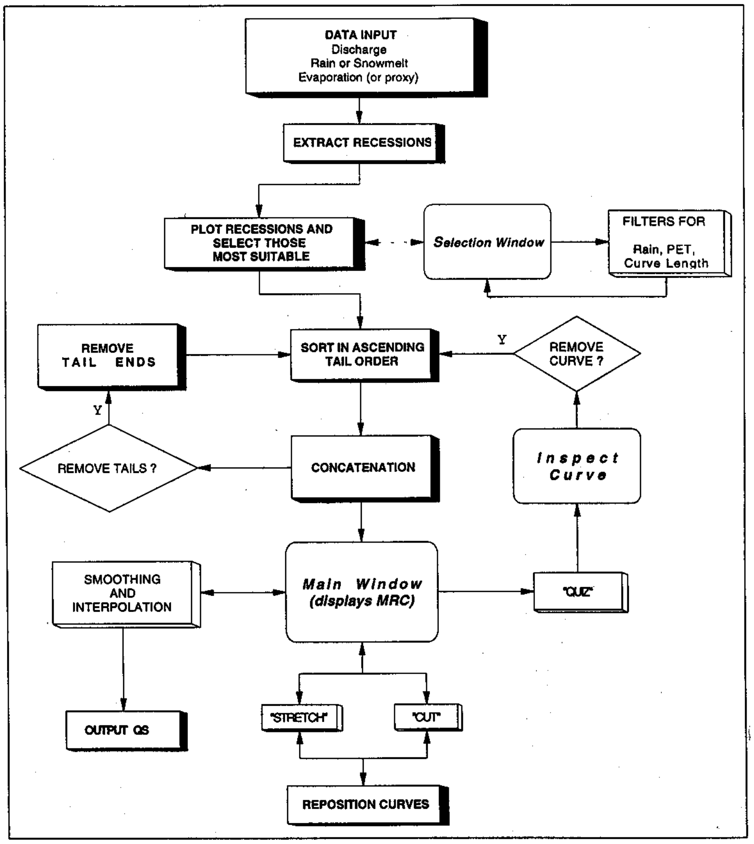

Fig. 1 *Schematic outline of the 'MRCtool' system.*

As the filter sliders are moved, recessions change colour on the screen to indicate whether or not they will be excluded from further analysis. The final choice is made by the operator and is necessarily subjective.

FORMATION OF THE MASTER RECESSION TURVE

Once a set of suitable recession periods has been selected, the next stage of the MRCtool procedure is to form a prototype master curve. This process can be referred to as *automated curve concatenation*, since the first estimate of the MRC is formed by merging individual recessions together on a common time axis, to obtain a continuous synthetic curve.

The concatenation algorithm is illustrated schematically in Figure 2. It works by sorting selected recessions into ascending order, based on lowest or 'tail-end' discharge. A simple procedure is then adopted in which recessions are considered in pairs, each recession being concatenated with the curve that has the next highest tail end discharge value. The lower curve of each pair is scanned upwards, so that concatenation occurs when the discharge on the lower curve is as near as possible to the discharge at the next recession tail-end. The remaining head of the lower curve may still be shown graphically, although it is no longer a part of the MRC, whilst the upper curve becomes the new lower recession for the next pairing.

The reason for adopting an upwards search procedure in the concatenation algorithm is that shifting the MRC from one recession to the next *at the lowest possible discharge* should tend to exclude storm flow effects from the master curve. This is essentially a heuristic means of avoiding explicit hydrograph separation. The success of the approach will depend on the quality and coverage of the data. If there are ranges of discharge for which no recession curves have been retained, then the concatenation algorithm will follow a lower curve up to its head and then jump directly to the next available recession tail. The result of this will be a conspicuous inflection in the MRC that may be treated using the interactive functions (described below).

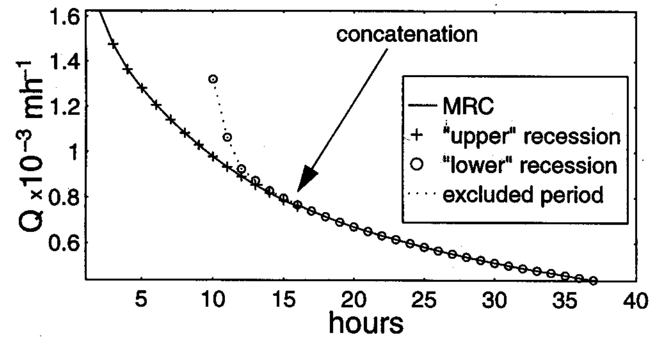

Fig. 2 *Matching two recessions, sorted in order of tail-end discharge, onto a master recession curve (MRC) (data from the River Wye, 1984). The dotted line indicates data excluded from the MRC by the upwards concatenation procedure, which 'scans' the data, as shown on this graph, from right to left.*

An alternative to the tail-up concatenation procedure would be to work from high to low discharges. This approach would make the MRC very sensitive to the initial exclusion of recession periods corresponding to surface runoff. As Boughton (1995) has noted, the timing of surface runoff is spatially variable and may be difficult to diagnose given areally–averaged rainfall data. The tail-up concatenation procedure is more sensitive to the exclusion of evaporation effects. These are also difficult to diagnose but may be somewhat more uniformly related to the proxy data that are normally available, such as estimates of PET.

The working MRC is displayed within an on-screen window, an example of which is shown in outline in Figure 3. Solid lines are used to plot the parts of each recession curve that form the MRC. Dotted lines show the remaining parts of each curve, to give an indication of the degree of variability of the recession curve data set around the MRC.

INTERACTIVE CURVE MATCHING

The interactive functions of the MRCtool system allow individual recession curves to be selected using a pointer on the screen, examined in detail, and then either removed from subsequent analysis or shifted in time relative to other recessions and cropped until a good alignment is found between curves. Three functions are currently available from the user interface, seen in Figure 3.

The QUIZ function generates detailed graphs of a selected recession and its associated evaporation and rainfall data. It also calculates totals for the curve of rainfall, evaporation, discharge and volumetric ratios of rainfall and evaporation to discharge, summed over the curve duration. An option is given to remove the selected curve entirely from the MRC. The QUIZ function is provided to allow a more thorough selection/rejection facility than can be provided by the automatic filters described above. If a given recession period appears to be unsuitable for inclusion within the MRC, for instance due to the apparent effect on discharge of a rainfall event near the tail of the recession, then that curve may be removed entirely from the recession data set and the working MRC reformed.

The STRETCH function allows a selected curve to be re-positioned relative to others on the time axis. The effect of this procedure is usually to stretch out the MRC, reducing its overall slope (the STRETCH procedure does not affect the slope of any individual curve, merely its position relative to others).

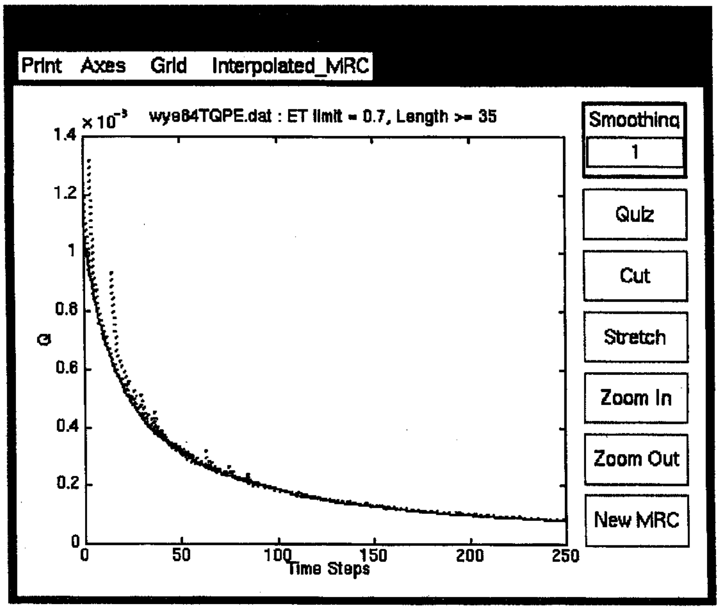

Fig. 3 The main interactive window in the MRCtool system showing the graphical user interface.

The CUT function allows the upper head of a given recession to be cropped. This is useful to correct for instances where a step has been produced in the MRC by the inclusion of the steep upper section of a recession. Data cut in this way remain visible on screen, plotted as dotted lines, but are not included within the MRC.

Using the interactive functions, data judged to be unrepresentative can be partially or entirely removed from the MRC. Storm flows will tend to be excluded from the MRC by the tail-up concatenation procedure. Where this fails, for instance if only one complete recession is available over a certain range of discharges, then the step-like anomaly remaining within the MRC may be removed using the CUT and STRETCH functions.

The interactive functions should allow some irregularities within the sampled recession data to be controlled. As Tallaksen (1995) points out, this type of manipulation may cause the MRC to be telescoped or contracted, although this may make little difference to the inferred underlying $Q\Delta S$ relationship if the slopes of each recession are matched consistently. Nevertheless, manipulation of the MRC is best avoided by the careful selection of representative recession periods covering a range of flows.

Smoothing the MRC and calculation of a discharge–storage relationship

Following adjustment by the user, the MRC data held by MRCtool consist of a set of recession curve segments located on an arbitrary time axis. A smoothing and interpolation algorithm is then be applied to these data. The smoothing algorithm currently used is an integrated random walk (IRW) procedure (Young *et al.*, 1991). The IRW algorithm makes no assumptions concerning the nature of the function underlying its input variable. The algorithm can interpolate across missing data but is not suitable for extrapolation. The degree of smoothing is controlled by a parameter which may be adjusted interactively.

Once an acceptable, smoothed MRC has been produced, a relationship describing the variation of discharge against relative storage deficit is calculated (under the assumption of zero recharge) by cumulatively summing discharge per unit time along the MRC. Figure 4 illustrates this procedure with an example MRC. The storage (S) or storage-deficit ($-S$) axis in the $Q\Delta S$ relationship is specified relative to an arbitrary datum, reflecting uncertainty about the transition from saturated zone discharge to non-saturated zone 'storm' flow.

The assumption of zero recharge is adopted for simplicity but it may also be possible to interpret the relative storage axis in terms of some effective storage, which takes saturated zone recharge into account, if this can be assumed to have a constant relationship to discharge. Either assumption is clearly a crude approximation to the complex continuity relationships in reality, but experience suggests that this does not necessarily result in poor results if the $Q\Delta S$ relationship is used to simulate catchment baseflow in a rainfall–runoff model (Lamb, 1996).

The summing procedure used to calculate the $Q\Delta S$ relationship is analogous to integration, but involves explicit time stepping. The procedure may therefore be sensitive to the time step length, suggesting that the time step chosen in any application of the calculated $Q\Delta S$

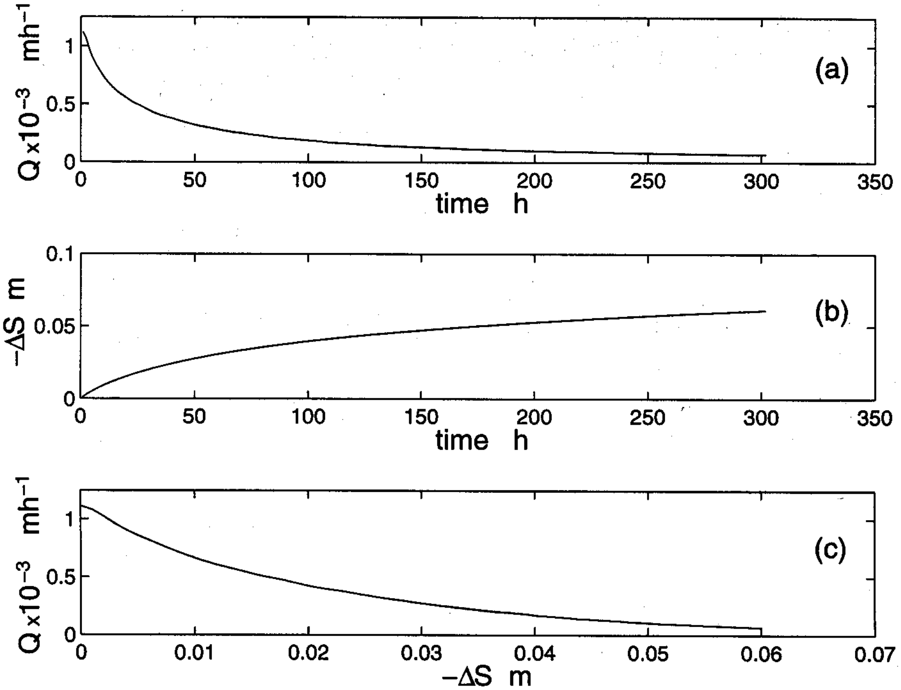

Fig. 4 *Graphs illustrating the calculation from an MRC of a discharge–relative storage ($Q\Delta S$) relationship for use as a saturated zone store. (a) MRC for the River Wye (see Figure 5, $L > 25$ hours.) (b) Derived changes in relative storage deficit, $-\Delta S$, over time, calculated such that the deficit associated with the peak discharge is arbitrarily chosen to be zero. (c) Derived $Q\Delta S$ relationship.*

relationship in tabular form should not be much smaller than the sampling interval of the source recession data.

If the MRC is taken to describe the response of a lumped catchment store, then its peak discharge should ideally represent the output of the store when at maximum capacity. However, the lumped baseflow storage model is perhaps least realistic for high flows, due to the likely influence of storm flow processes at higher discharges. This may not be a problem if an MRC is used to define the baseflow component within a catchment rainfall–runoff model, since the upper ranges of the MRC may rarely be approached by the saturated zone discharge during simulations. The correct specification of the shape of the mid and lower range of the MRC may be of more importance in modelling flows over longer periods.

Examples

Three sites were chosen for example applications of the MRCtool system. The first site, an Institute of Hydrology research catchment on the River Wye (UK), was chosen, since it has already (Quinn and Beven, 1993) been studied using the catchment model TOPMODEL, assuming an exponential conceptual saturated zone store. The second catchment, the Ringelbach in France, has also been investigated using TOPMODEL concepts by Ambroise *et al.* (1996a,b). This catchment was chosen because of existing work by Ambroise (1988) who used recession curve analysis to show that a parabolic saturated zone storage function was appropriate there. The third site is the Seternbekken MINIFELT catchment in Norway, which has been studied using a generalised form of TOPMODEL, based on the $Q\Delta S$ concept, by Lamb (1996).

THE RIVER WYE

The MRCtool system has been applied to hydrological data for 1984 from the River Wye research catchment (10.55 km^2), which drains part of the Plynlimon massif in Wales (Kirby *et al.*, 1991, Hudson and Gilman, 1993). The exponential store, as used in TOPMODEL by Quinn and Beven (1993), may be written:

$$Q = Q_0 e^{S/m} \qquad (1)$$

where S is the catchment–averaged storage, Q_0 is the zero–storage discharge and m is the exponential function shape parameter.

Quinn and Beven (1993) found that different values of m were obtained by calibration on wet and dry periods. Overall, TOPMODEL was able to simulate flows over periods of up to nine months, with simulation efficiencies (using the statistic of Nash and Sutcliffe (1970)) of over 85% being obtained for the calibration periods. The good results obtained by Quinn and Beven (1993) suggest that the exponential form of Equation (1) is a suitable description of the storage characteristics of the Wye saturated zone.

Hourly observations of discharge, rainfall and PET for the year 1984 were analysed using the MRCtool system. 784 individual recessions were extracted from the flow record, the longest being of 110 hours duration. These data were filtered to reject any recession curves for which the total volume of coincident PET or rainfall was greater than 10% of the volume discharged during the recession. This left 66 curves suitable for further analysis.

Using the reduced set of recession curves, the automated concatenation algorithm was applied to construct four master recessions, where the minimum lengths of the component recession periods were specified as 90, 50, 25 and 5 hours. These MRCs are shown in Figure 5 where the minimum length of the component curves is shown by the letter L.

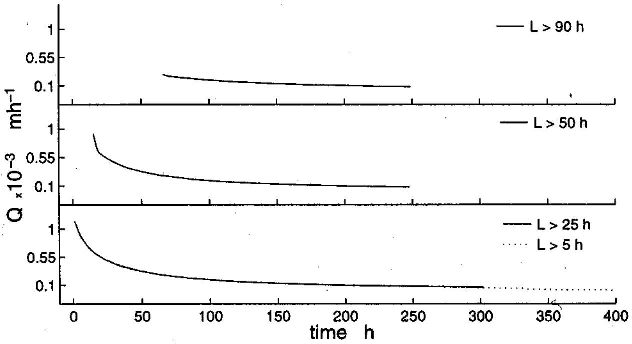

Fig. 5 *MRCs produced automatically by MRCtool using 1984 data from the River Wye.*

Relationships were calculated from each MRC between discharge and $-\Delta S$, the storage deficit relative to an (arbitrarily) assumed deficit of zero at the maximum flow. These $Q\Delta S$ relationships are shown in Figure 6, where discharge is plotted on a logarithmic axis. The relationships plot as straight lines for $Q > 7 \times 10^{-5}$ mh^{-1}, which supports the interpretation that an exponential store is an appropriate form for the Wye catchment.

As the limit on the duration of the component recessions was reduced from 90 to 5 hours, the number of recessions included in each MRC naturally increased, bringing more recessions at higher flows into the MRC and thus shifting the intercepts in Figure 6. Despite this, the lines in Figure 6 appear for the most part to be consistent in slope. The $Q\Delta S$ relationship derived from recessions of greater than 25 hours' length becomes steeper at high flows; this is due to the uppermost recession in the MRC and is attributable to a rainfall-induced storm response. Only when recessions were allowed to be as short as 5 hours were flows of less than 7×10^{-5} mh^{-1} observed in the MRC. However, the exponential $Q\Delta S$ relationship seems to break down for these data.

Equation (1) may be written;

$$\ln Q = \frac{1}{m} S + \ln Q_0 \tag{2}$$

allowing m to be estimated from the gradients of the lines in Figure 6. Table 1 gives estimates of m taken from these $Q\Delta S$ relationships and also the calibrated values obtained by Quinn and Beven (1993).

Table 1. Estimates of *m* (in metres), using MRCtool compared with the TOPMODEL calibrations of Quinn and Beven (1993).

	Min. length (hours)	Number of curves	*m* parameter (metres)
MRCtool	90	2	0.0214
	50	5	0.0213
	25	18	0.0218
	5	58	0.0219
TOPMODEL using exponential store	dry period calibration		0.012
	wet period calibration		0.0079

The estimates for m given by Quinn and Beven (1993) are smaller than those obtained using MRCtool. Smaller values of m lead to a steeper recession curve and may have been obtained during model calibration due to the sensitivity of the efficiency measure used by Quinn and Beven (1993) to the correct fitting of peak flows. The simulations performed by Quinn and Beven (1993) comprised over 85% subsurface flows. It seems likely therefore, that some relatively steep parts of the observed recession data excluded from the MRCtool analysis as probable rainfall-induced storm flows, were in fact modelled by Quinn and Beven (1993) as saturated zone discharge, leading to a steeper calibrated recession.

LOW FLOW ANALYSIS

In Fig. 6, it can be seen that flows of less than 7×10^{-5} mh^{-1} appeared in the MRC only when relatively short recession curves were included. However, these short, low flow recessions caused the inferred exponential $Q\Delta S$ relationship to break down because of a failure of the automatic concatenation algorithm caused by sparse coverage of discharges in the sampled recession data. Figure 7(a) illustrates the problem by showing the concatenated MRC on semilogarithmic axes to emphasise the low flows. It can be seen that, although there are individual recessions of consistent slope at low flows,

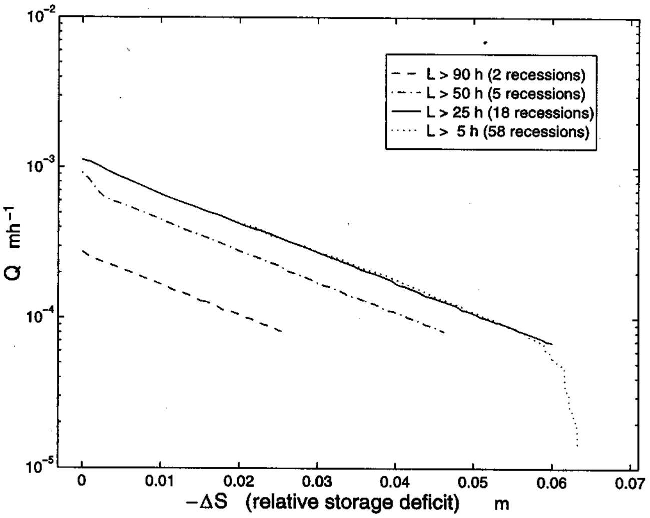

Fig. 6 *$Q\Delta S$ relationships produced automatically by MRC-tool using 1984 data from the River Wye.*

these do not cover a sufficient range of discharges to be reliably matched by eye onto a master curve.

Relaxing the selection criterion for PET allowed a wider range of low flow recessions to be investigated. These are shown in Fig. 7(b) (note the change to arithmetic axes). There appear to be two dominant modes of behaviour in these curves, a flat upper recession followed by a steep draw-down. These recessions are not influenced by rainfall-induced flows since the data continued to be filtered to remove periods coinciding with rainfall. The abrupt changes in slope must therefore be due to evaporation.

Comparison of the PET and discharge records suggested that the recessions shown in Figure 7(b) reflect a diurnal cycle of evaporation losses (the steep drawdowns) which are superimposed upon the master recession. During the night, the discharge recovers and then goes into storage-controlled recession again. A similar effect has been noted by Ambroise (1988), who found daily downward fluctuations in discharge for the Ringelbach catchment, due to evaporation from a small, almost constantly saturated area near the channel. The recovery from these cycles could deviate progressively over time from the master recession curve.

For the Wye data of 1984, it would appear that each day's recovery in discharge returned to a master recession. This hypothesis was tested in the MRCtool system by implementing a filter, based on a calculation of the recession curve gradient, to remove the steep, daytime components of each low flow recession. An MRC was automatically concatenated from these data and the derived $Q\Delta S$ relationship is shown in Figure 8, labelled 'auto'. The MRC contained a number of discontinuities, similar to those seen in Figure 7, but of lesser extent. Since the slopes of each recession segment were very consistent, the MRCtool interactive functions could easily be used to correct these discontinuities. The resulting, corrected $Q\Delta S$ relationship is also shown in Figure 8 and can be seen to conform closely to the exponential relationship identified for $Q > 7 \times 10^{-5}$ mh^{-1}.

To test this recession analysis, a low flow recession period, selected from the 1984 Wye data, was simulated using exponential stores where Q_0 was assumed to correspond to the initial discharge for which the relative storage deficit was set to zero. Using Equation (2), m was estimated from the $Q\Delta S$ relationships shown in Fig. 8. For $Q > 1 \times 10^{-4}$ mh^{-1}, m was estimated to be 0.024 m

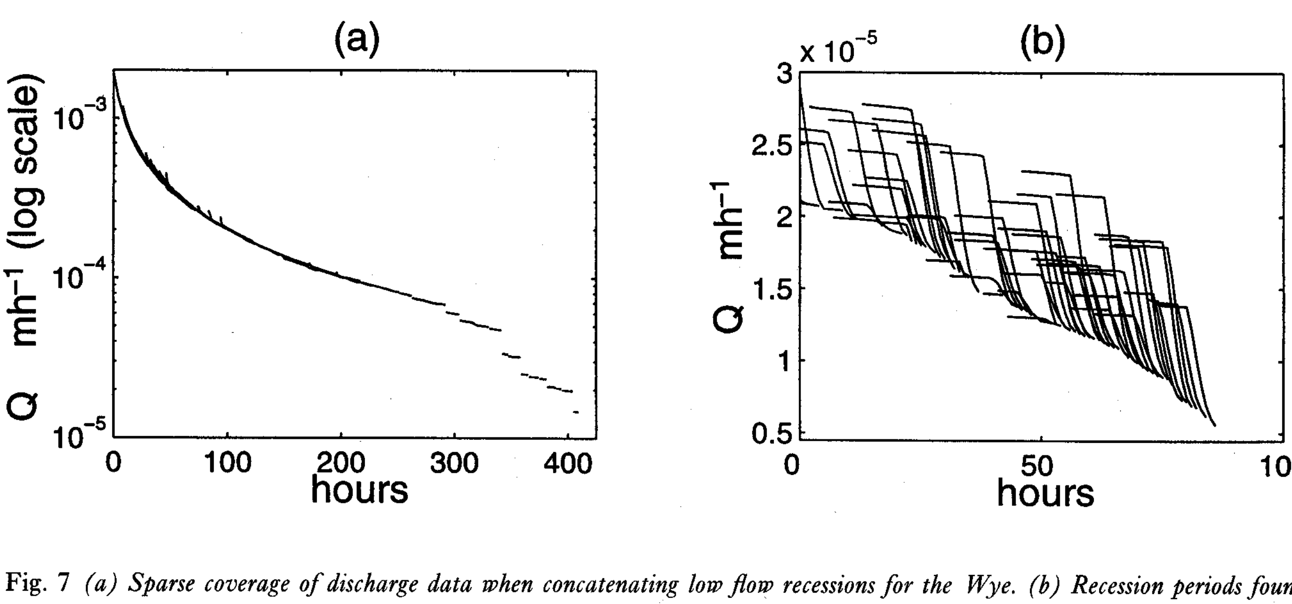

Fig. 7 *(a) Sparse coverage of discharge data when concatenating low flow recessions for the Wye. (b) Recession periods found under a relaxed PET limit.*

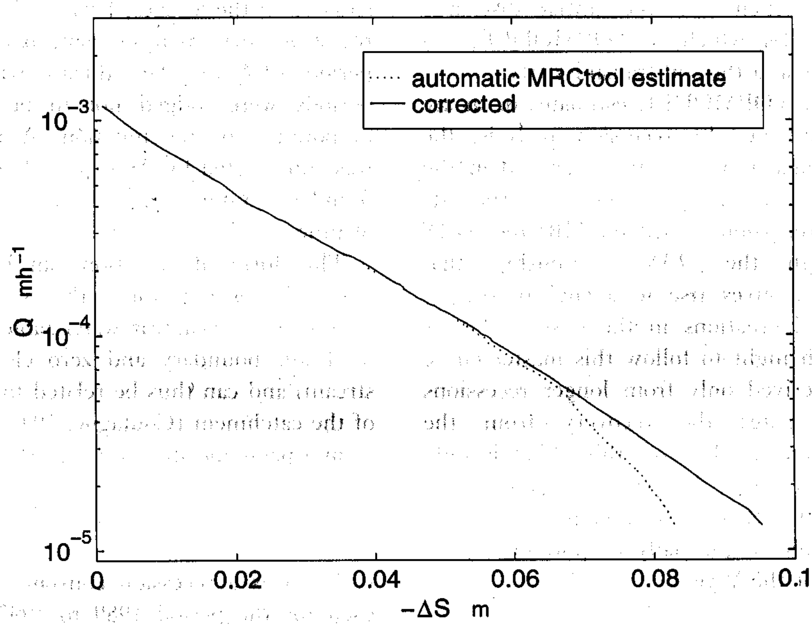

Fig. 8 *$Q\Delta S$ relationships for the Wye including low flow recessions (automatic and manually corrected cases shown).*

and for the corrected $Q\Delta S$ relationship that takes into account the low flow data, $m = 0.0215$ m.

Figure 9 shows the recessions obtained from these exponential stores compared with the observed discharges for the selected low flow period. Also shown, for comparison, are recessions obtained using estimates of m, from the TOPMODEL study of Quinn and Beven (1993). Note that these are included merely to illustrate the effect of changing the value of m under the assumption of a relative storage deficit of zero at the start of the recession. In a continuous TOPMODEL simulation, the period shown would be simulated differently, since

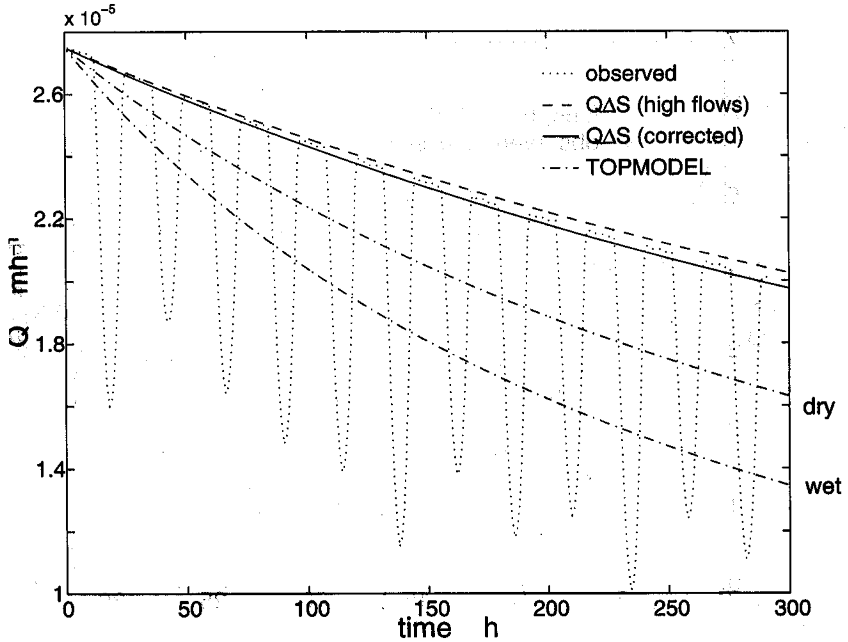

Fig. 9 *Low flow recessions simulated by exponential storage models derived using the MRCtool system (labelled $Q\Delta S$) and TOPMODEL calibrations for 'wet' and 'dry' periods.*

the start of the recession in Fig. 9 would correspond to some actual storage deficit and the initial discharge would not be equal to Q_0, which, in TOPMODEL, is a function of topography and the soil transmissivity.

Since the calibrated TOPMODEL estimates of m are smaller than those obtained by recession analysis, the TOPMODEL recessions in Fig. 9 are steeper than the curves derived by recession analysis. There is little difference between the recessions based on MRCtool $Q\Delta S$ relationships, although the $Q\Delta S$ relationship that includes low flow data gives rise to a slightly steeper recession. The daily fluctuations in the observed flows appear to recover each night to follow this master curve closely. The MRC derived only from longer recessions progressively overestimates the recovery from the observed daily fluctuations. However this effect is only very slight, even after a twelve day period as shown here. It appears, therefore, that there is little or no progressive depletion of storage due to the daily evaporation cycles apparent in the data for the Wye.

THE RINGELBACH CATCHMENT

Consistent estimates of the exponential store shape parameter *m* were obtained for the Wye catchment using automated algorithms. This is in part a reflection of the consistency of the Wye data, which record many winter hydrographs where the effects of evaporation are limited. Greater difficulties would be expected in analysing recessions from a catchment where the effect of evaporation is much stronger. One such catchment is the Ringelbach, situated in the Vosges, France. Ambroise (1988) reported recession curve analysis using hydrological data from the period 1976 to 1986, during which only 21 recession periods were judged not to be unduly influenced by evaporation or precipitation. A master recession curve was constructed by Ambroise (1988) from these data and found to conform to a second order hyperbolic function of time.

This form of recession can be obtained analytically from the groundwater theory of Boussinesq (1904, assuming a curvilinear water table, impermeable horizontal lower boundary and zero elevation of water in the stream) and can thus be related to physical characteristics of the catchment (Coutagne, 1948). It can also be derived from a parabolic storage function;

$$Q = Q_0(1 + \alpha S)^2 \tag{3}$$

where α is a recession constant. Equation (3) has been used for the period 1989 to 1992 in a modified TOPMODEL applied to the Ringelbach catchment (Ambroise *et al.*, 1996b) where the value of α was specified from the master recession curve.

The MRCtool system was used to identify an MRC for the Ringelbach, given daily data for the years 1989 to 1992, and to compare this to the established recession function. The MRCtool concatenation algorithm could not generate a smoothly matched master curve until the maximum permissible ratio of PET to storage change per

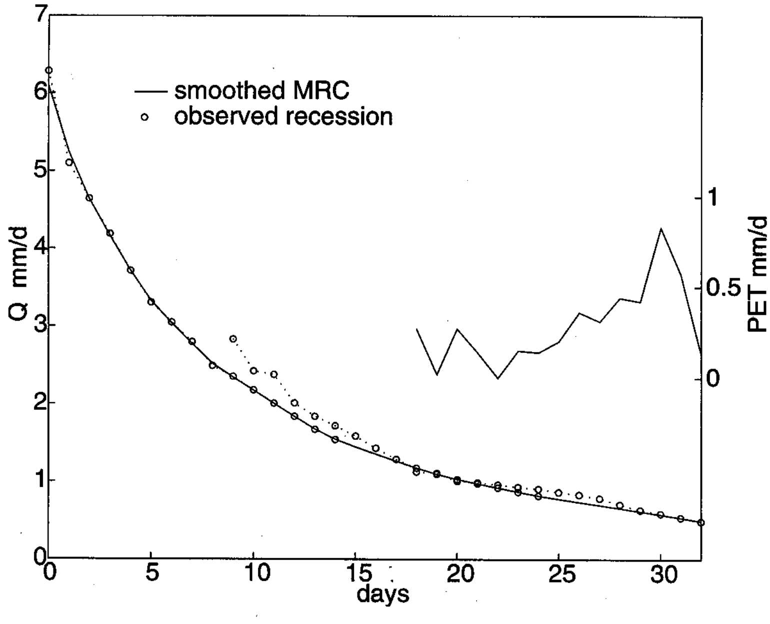

Fig. 10 *MRC for the Ringelbach catchment, obtained using MRCtool with daily data from the years 1989 to 1992.*

recession was increased to 30%, the maximum ratio for rainfall was increased to 50% and recessions of less than 14 days duration were rejected. Three periods were found that satisfied these conditions, although one of these appeared to have been influenced by precipitation beyond the first day.

These recessions were matched to form an MRC, (Fig. 10), where observed recession data are represented by circles and a smoothed master curve has been fitted through these data by the IRW smoothing algorithm. One intervention was required by the operator to exclude flows that had been affected by rainfall between days 10 and 16 of the MRC. Also shown in Figure 10 is the PET series for the lower of the three component recessions. There is an increase in PET during this recession period and this appears to cause the observed recession to steepen towards its end.

From Equation (3) it follows that $Q^{1/2}$ will plot as a straight line against $-S$ or $-\Delta S$ for the parabolic storage function. The $Q\Delta S$ relationship derived using the MRCtool system is shown in this form in Figure 11 and compared to a parabolic $Q\Delta S$ relationship derived from the master recession function of Ambroise (1988). The MRCtool $Q\Delta S$ relationship approximates closely to the parabolic function and is of similar slope.

This example reinforces the importance of the selection of suitable data, in this case because evaporation has a great effect at low flows in the Ringelbach. Only when suitable data can be found over a range of flows that is characteristic of the catchment can an MRC be used to infer with confidence an underlying analytical function. Using automated selection procedures on four years of data, an MRC has been identified only for the range of discharges $0.5 > Q > 6.1$ mm day^{-1}. Ambroise (1988) was able to find a greater range of suitable data from an eleven year period and showed that the parabolic recession function was appropriate to discharges of below 0.15 mm/day.

However, a first estimate for a catchment $Q\Delta S$ relationship may be obtained even from a relatively limited dataset of recession curves that are less than ideally representative of 'pure' saturated zone drainage. This is illustrated in the following section.

THE SETERNBEKKEN MINIFELT CATCHMENT

As a final example of the use of the MRCtool system, data from the Seternbekken MINIFELT catchment in Norway were analysed. This small (7500 m^2) catchment has been described by Myrabø (1988) and Erichsen and Myrabø (1990). Hourly data from a snow-free period of 9 weeks in the autumn of 1987 were analysed using MRCtool. Only 6 periods of any length were found under the condition that the volume of PET should not exceed 10% of the storage change during any given recession. The selected recessions were matched together interactively, resulting in the $Q\Delta S$ relationship shown in Figure 12, where discharge is log transformed.

No clear form can be identified for $Q < 3.4 \times 10^{-4}$ mh^{-1} where the $Q\Delta S$ relationship derives from a single recession which becomes steep towards its end due to the influence of evaporation. For $Q > 3.4 \times 10^{-4}$ mh^{-1} it would appear that an exponential function would be appropriate to describe the $Q\Delta S$ relationship. However, it is difficult to interpret this finding as a general analytical storage function for the catchment, since the maximum discharge in Fig. 12 ($Q = 1.9 \times 10^{-3}$ mh^{-1}) is considerably less than the greatest flow in the record.

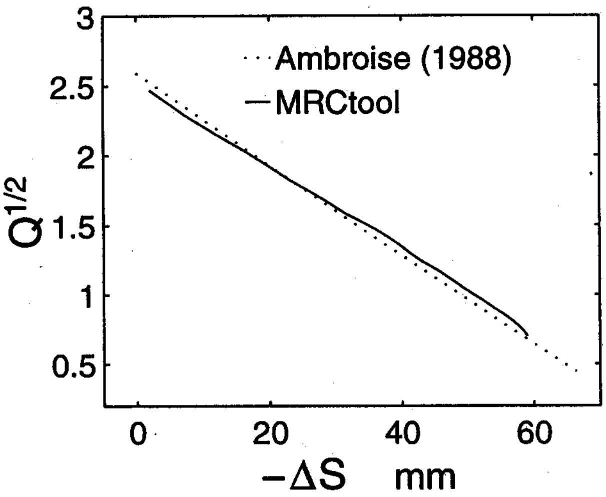

Fig. 11 *$Q\Delta S$ relationship for the Ringelbach, derived by applying MRCtool to data from the period 1989–1992, compared with the analytical storage model of Ambroise (1988). (Q in units of mm/day.)*

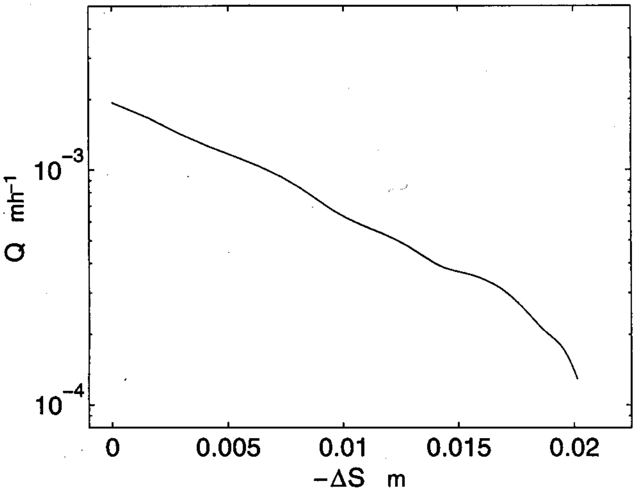

Fig. 12 *$Q\Delta S$ relationship ($Q < 1.9 \times 10^{-3}$ m/h) obtained for the Seternbekken MINIFELT catchment, rejecting recessions associated with rainfall.*

To extend the analysis, the limit of a 10% rainfall to discharge ratio was increased to 50% to include recessions at higher flows. This resulted in the inclusion of a long recession period (over 100 hours) that originated at the peak discharge recorded in the source data set, $Q = 7.9 \times 10^{-3}$ mh^{-1}. This recession was matched into the existing MRC, thus extending the upper limit of the $Q\Delta S$ relationship. The peak of this recession is strongly associated with rainfall, but, from visual inspection, flows below $Q \approx 4 \times 10^{-3}$ mh^{-1} did not appear to be.

The revised $Q\Delta S$ relationship is shown in Fig. 13 where Q is plotted on an arithmetic axis. This graph suggests that the storage characteristics of the catchment are approximately linear for $Q > 2 \times 10^{-3}$ mh^{-1}, but that an exponential function may be appropriate for discharges below this value. There is a risk that the linear relationship represents, at least in part, rapid, rainfall-induced surface and near-surface flows. However, observations in the catchment have shown that the water table rises to the surface extensively at these higher discharges. The flows represented in the linear part of the derived $Q\Delta S$ relationship may therefore arise from a combination of rain falling directly onto saturated areas and exfiltration, or 'return flow' from the saturated zone.

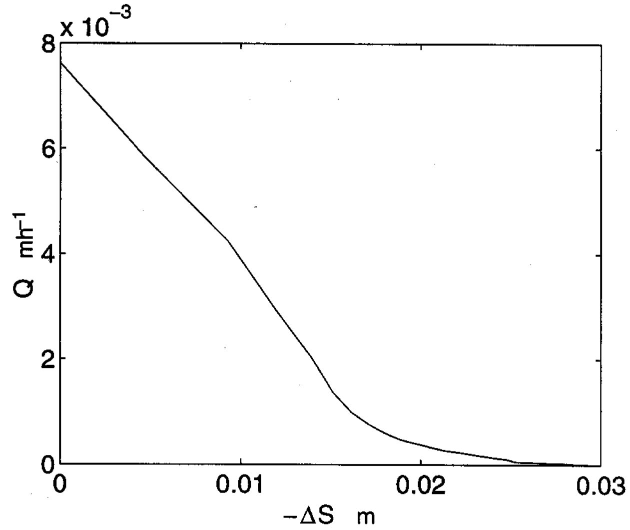

Fig. 13 *$Q\Delta S$ relationship obtained for the Seternbekken MINIFELT catchment, by relaxing the limit on the volume of rainfall associated with each recession period.*

Conclusions

It has been shown, using the River Wye data, that an automated system can be used to estimate, with some consistency, the saturated zone storage characteristics of a catchment, given sufficient observations of recession periods free from the effects of evaporation. However, where evaporation exerts a stronger influence on recorded flows, it may be difficult to find acceptable recession periods.

Given the concatenation procedure used in this study, the selection of curves associated with low potential evaporation, especially for low flows, is critical. Ambroise (1988) presented illustrations of the deviations from the Ringelbach master recession of flows affected by evaporation. Such deviations can be considerable and, if not removed, would result in a marked over-steepening of the MRC generated by MRCtool. Unfortunately, low flow periods often coincide with periods of high potential evaporation.

In the Ringelbach case, a few useful recession periods could be found using automated filters. These data allowed an underlying recession or storage function to be inferred over a limited range of flows. To identify the master recession more conclusively, a much longer period of the hydrological record has to be examined to find suitable recessions.

Ideally, a large sample of recession data is required from which to select with care suitable recession periods for MRC analyses. However, as long as there are at least a few recessions not greatly affected by evaporation or precipitation, then a semi-automated system, such as the MRCtool package, may be used to match these data together interactively and to remove from the MRC any parts of an individual recession that appear to be unsuitable. If further data become available, then the estimated $Q\Delta S$ relationship may be improved.

In the Seternbekken example, a $Q\Delta S$ relationship was found that did not conform well to any single analytical function. This example also brings into question the conceptual definition of the catchment saturated zone store. At the upper limits of the available recession data, the problem of distinguishing between storage effects and surface storm flows has to be faced (essentially the 'baseflow separation' problem).

The Seternbekken example shows that a system such as MRCtool may be used to estimate an appropriate storage function for a catchment given even a relatively limited set of flow measurements. However, it may be that the $Q\Delta S$ relationship obtained from these data does not conform well to any single analytical function. In this case, the addition of extra data changed the estimated form of the catchment storage function from an approximately exponential relationship to a compound linear/exponential form. As an empirical relationship, this compound function adequatly represents the storage characteristics of the catchment in a semi-distributed rainfall–runoff model, at least over the range of flows commonly observed (Lamb, 1996).

To some extent, the saturated zone discharge–storage relationship inferred by recession curve analysis for a catchment will depend upon the underlying conceptual model. The problem of identifying a zero absolute storage deficit condition is particularly difficult. However, experience from previous modelling studies suggests that simple storage relationships may be adequate to describe the main features of a catchment's hydrological responses, especially in the light of other sources of error, such as measurement errors and the over-simplificaton of internal hydrological processes and their spatial structures.

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