A parameterization of momentum roughness length and displacement height for a wide range of canopy densities

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Abstract

Values of the momentum roughness length, $z_0$, and displacement height, $d$, derived from wind profiles and momentum flux measurements, are selected from the literature for a variety of sparse canopies. These include savannah, tiger-bush and several row crops. A quality assessment of these data, conducted using criteria such as available fetch, height of wind speed measurement and homogeneity of the experimental site, reduced the initial total of fourteen sites to eight. These data points, combined with values carried forward from earlier studies on the parameterization of $z_0$ and $d$, led to a maximum number of 16 and 24 data points available for $d$ and $z_0$, respectively.

The data are compared with estimates of roughness length and displacement height as predicted from a detailed drag partition model, R92 (Raupach, 1992), and a simplified version of this model, R94 (Raupach, 1994). A key parameter in these models is the roughness density or frontal area index, $\lambda$.

Both the comprehensive and the simplified model give accurate predictions of measured $z_0$ and $d$ values, but the optimal model coefficients are significantly different from the ones originally proposed in R92 and R94. The original model coefficients are based predominantly on measured aerodynamic parameters of relatively closed canopies and they were fitted 'by eye'. In this paper, best-fit coefficients are found from a least squares minimization using the $z_0$ and $d$ values of selected good-quality data for sparse canopies and for the added, mainly closed canopies.

According to a statistical analysis, based on the coefficient of determination $(r^2)$, the number of observations and the number of fitted model coefficients, the simplified model, R94, is deemed to be the most appropriate for future $z_0$ and $d$ predictions. A $C_R$ value of 0.35 and a $C_{d1}$ value of about 20 are found to be appropriate for a large range of canopies varying in density from closed to very sparse. In this case, 99% of the total variance occurring in the $d$-data across 16 selected canopies can be explained, whereas the analogous value for the $z_0$-data (24 data points available) is 81%. This makes the R94 model, with only two coefficients and its relatively simple equations, a useful universal tool for predicting $z_0$ and $d$ values for all kinds of canopies.

For comparison, a similar fitting exercise is made using simple linear equations based on obstacle height only (e.g. Brutsaert, 1982) and another formula involving canopy height as well as roughness density (Lettau, 1969). The fitted Brutsaert equations explain 98% and 62% of the variance in the $d$- and $z_0$-data, respectively. Lettau's equation for prediction of $z_0$ performs unsatisfactorily $(r^2$ values <0, even after fitting of the coefficient) and so it is concluded that the drag partition model is definitely the most effective for prediction of the momentum roughness lengths for a wide range of canopy densities.

Introduction

Models of momentum transfer to the ground describe the surface in terms of two key parameters—the aerodynamic roughness length, $z_0$, and the zero plane displacement height, $d$. These parameters are usually found by fitting the logarithmic wind profile equation to measured wind profiles. Their values are then assigned to the surface and used in subsequent calculations, such as of
land-atmosphere interactions in a climate model. It is inconvenient to measure wind profiles over all the surfaces considered in a large-scale model so, naturally enough, there have been attempts to relate $z_0$ and $d$ directly to measurable surface properties.

Early attempts were made to express $z_0$ and $d$ as simple fractions of the vegetation height (see Brutsaert, 1982). Typically, it was found $z_0/h \sim 0.13$ and $d/h \sim 0.66$. These relationships were based predominantly on data obtained in humid areas for agricultural crops and forests covering most of the ground area. However, it is clear that plant height cannot be the only vegetative characteristic involved and measured values often depart considerably from these norms, particularly for sparse canopies and row crops. As examples, Garratt (1980) found $z_0/h = 0.05$ for a sparse savannah vegetation while Hatfield (1989) found $z_0/h = 0.5$ for a cotton canopy. In these cases, other factors such as plant spacing or foliage area density must have a role.

Sparse canopies are found mostly in areas governed by a (semi-) arid climate which cannot support full vegetative cover. Recently, climatological interest has shifted to these drier areas and several experiments can be mentioned in this context including FIVE (Sellers et al., 1988), HAPEX-Mobility (André et al., 1986), SEBEX (Wallace et al., 1992), EFEDA (Bolle et al., 1993), MONSOON (Kustas and Goodrich, 1994) and HAPEX-Sahel (Goutorbe et al., 1994). More insight is needed into the considerable influence of these semi-arid areas on global circulation, triggered by growing concern about the greenhouse effect and desertification (ICIII, 1986; Hare and Ogall, 1993). Hence, there is a need for reliable values of surface parameters, such as roughness length, representative for these sparse canopies.

More sophisticated models have been drawn up to describe $z_0$ and $d$ for sparse canopies. Most of these models are based on a partitioning of total drag, $\tau$ (kg m$^{-1}$ s$^{-2}$) into canopy and ground components, so they apply to both closed and sparse canopies. Drag partitioning was introduced by Schlichting (1936) and it was tested by Marshall (1971) using wind tunnel experiments. Follow-up studies were made by Wooding et al. (1973), Seginer (1974), Arya (1975) and Kondo and Akashi (1976). Since then, Raupach (1992) (referred to here as R92) has developed a simple analytical treatment of drag partition theory based on scaling and dimensional analysis. In 1994, he published a simplified version of his model (Raupach, 1994) (R94), which involved fewer independent variables and coefficients. Corrigenda to R92 and R94 appeared in Raupach (1995).

A key parameter in Raupach’s drag partition model is the roughness density or frontal area index, $\lambda(-)$, defined by

$$\lambda = bh/D^2,$$

which was introduced by Lettau (1969) and formally justified by Wooding et al. (1973). Here, $h(m)$ and $b(m)$ are the roughness element height and breadth, and $D(m)$ is element spacing.

In this paper literature values of $z_0$ and $d$ measured over a number of sparse canopies and row crops are extracted. The quality of these values is assessed, taking into account site characteristics and instrument locations. The most reliable of them are retained for comparison with the theoretical estimates and they are combined with several other well-tested $z_0$ and $d$ values, found mainly for closed canopies, also from the literature. Next, the ability of Raupach’s drag partition model is tested. Raupach’s model, in its full and simplified versions, was originally calibrated using mostly measurements from relatively closed real canopies ($\lambda \gg 0.5$). It will be tested here to find whether it can also predict roughness parameters for sparser canopies.

For comparison, the original Lettau (1969) equation which relates $z_0$ directly to $\lambda$, and the simple relationships using canopy height only (e.g. Brutsaert, 1982) are also tested on the data. For all parameterizations, model coefficients will be calculated using a least squares minimization procedure and the models’ skills to describe the data will be compared.

**Material and Methods**

**CRITERIA FOR RATING THE QUALITY OF EXPERIMENTAL ROUGHNESS PARAMETERS**

**Limits of the logarithmic wind profile equation**

All methods used to determine $z_0$ and $d$ make use of the logarithmic wind profile (Tennekes, 1973), relating wind-speed, $u$ ($m s^{-1}$), at a level $z$, to the friction velocity $u^*$ ($m s^{-1}$):

$$u = \frac{u^*}{k} \left[ \ln \left( \frac{z-d}{z_0} \right) - \Psi_m\left( \frac{z-d}{L} \right) \right],$$

(2)

where $\Psi_m(-)$ is the integrated stability function with $L$ (m) the Obukhov length and $k$ is the von Kármán constant.

Eq. (2) describes the wind profile above the roughness sublayer, when the air flow is in equilibrium with a level, homogeneous surface. If field data are to be analyzed using Eq. (2), they must be from instruments located above the roughness sub-layer, high enough to ensure that they are not influenced by the rather different turbulence-generating processes operating near the canopy top. Furthermore, the instruments must be located at a height that will ensure sufficient fetch over uniform ground.

**The depth of roughness sublayer, $z^*$**

Early on, the depth of the roughness sublayer was calculated as a multiple of canopy height, which is a practical criterion for low-concentration surfaces (Raupach et al.,
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1980). A multiplication factor of 2.0 has been quoted by O'Loughlin and Annambhotla (1969), while Garratt (1978) used a value of 4.5. Later, the idea developed that non-logarithmic behaviour close above vegetation was caused by 'the action of turbulent wakes generated by the flow around the roughness elements' (Garratt, 1980). From this arose the view that the depth of the roughness layer is described more adequately by the spacing of the roughness elements (see Garratt, 1980; Raupach et al., 1980; Chen Fazu and Schwerdtfeger, 1989). Recently, ideas have changed again. It has now been shown that the turbulence-generating processes near the top of many plant canopies bear a strong resemblance to those operating about the velocity inflexion in a mixing layer (Raupach et al., 1989; Raupach et al., 1996; Brunet, 1996).

The vertical length scale for the active canopy turbulence, $L_v$, is given by $u_h/(du_h/dz)$ where $h$ is canopy height, $u_h$ is mean velocity at $h$ and $(du_h/dz)$ is measured at the inflexion of the velocity profile, located near the top of the canopy. The influence of this turbulence might be expected to extend above the canopy top by a distance which scales on $L_v$, by analogy with mixing layers. That is, $z^* = h + cL_v$, where the constant $c$ has a value which is probably about two or three.

Measurements to calculate $L_v$ are rarely available. It is likely that $z_0$ is proportional to $L_v$. A very preliminary analysis (Raupach, pers. com. 1996) suggests that $L_v \approx 6z_0$ so $z^* \approx h + 15z_0$ is as good a guess as can be managed at present. This relationship bears a strong similarity to the relationship proposed by De Bruin and Moore (1985), viz. $z^* = 20z_0 + d$, (3)

so either expression will do for the data screening.

Fetch
Fetch is the distance the wind passes over uniform ground before reaching an observation point. Fetch is adequate for use of Eq. (2) when it is long enough for the wind flow to come into equilibrium with the ground up to at least the height of the highest wind instruments. A commonly-used rule of thumb is that this occurs when fetch is at least one hundred times the height of the highest anemometer. However, this criterion ignores the well-known dependence of the rate of equilibration on the roughness of the underlying surface (e.g. Gash, 1986). A formula which takes this dependence into account is given by Wieringa (1993):

$$F \approx 20z_0 \ln \left( \frac{10z}{z_0} - 1 \right),$$

where, $z$ is height of the top instrument and $F$ is the minimum fetch requirement. This formula takes no account of atmospheric stability, which will lead to an overestimation of the required fetch during unstable conditions and underestimation in stable conditions. This could be taken into account (see Lloyd, 1995), but Eq. (4) is used here to screen the data, because measures of stability are not easily deduced for the experiments described in the literature.

The adequacy of fetch in each experiment has been checked by comparing the fetch available at each site with the minimum adequate fetch calculated from Eq. (4), using the experimentally-determined roughness length in each case.

SUMMARY OF EXPERIMENTS AND ASSESSMENT OF DATA QUALITY

Raupach (1992, 1994) has collated data from a number of experiments, where experimental values of $z_0$, $d$ and $\lambda$ are available. Of these, the majority of the data for relatively sparse 'canopies' ($\lambda < 0.5$) are from experiments in wind tunnels (O'Loughlin, 1965; Raupach et al., 1980). Here, the data set has been extended by collating data from field sites with sparse canopies.

In the literature, 14 determinations of $z_0$ and $d$ from field sites with sparse vegetation have been identified. Five of these sites had savannah-type vegetation with undergrowths of sparse forbs and/or grass, seven were row crops (including vineyards) with bare soil between, two were tiger-bush with elongated patches of small trees and lower vegetation interspersed by bare soil, and two were from orchards with grass covering the ground. Two sets of data are included for each of two sites because they had two different values of $z_0$ depending on wind direction (R5), or leaf area index of the overstorey canopy (R7). From these reports, the 14 values of $z_0$ and $d$, have been extracted along with the relevant vegetation characteristics (height $h$, breadth $b$, spacing $D$, and canopy density $\lambda$). These are reported in Table 1.

Also in Table 1 are the quality ratings for each determination. Fetch was rated by assigning a '+' when the experimental fetch exceeded minimum requirements as given by Eq. (4), a '0' when fetch was close to the minimum, and a '-' when fetch was less than half the calculated minimum requirement. Sometimes the assessed fetches were stated explicitly in the references, while in two cases it was given as 'several kilometers' (S2, S3). In two cases, the fetch varied with wind direction (S4 and R5) and, in these cases, all the different experimental fetches had to be larger than the calculated fetch. Homogeneity of the experimental sites was judged qualitatively as good (+) or moderate (0) depending mainly on the variety of surrounding surface types.

The height of the profile instruments was judged to be good (+) when all $u$-levels were higher than $z^*$, poor (-) when no levels were above $z^*$ and moderate (0) when two or three levels were above $z^*$ as calculated by Eq. (3).

For the final quality rating, good (+) is given only to
Table 1. Summary of canopy and aerodynamic characteristics plus quality rating of 14 bush-type sparse canopies. The ‘+’, ‘0’ and ‘−’ signs denote good, moderate and poor quality, respectively.

<table>
<thead>
<tr>
<th>Vegetation type</th>
<th>( h ) [m]</th>
<th>( D ) [m]</th>
<th>( b ) [m]</th>
<th>( \lambda )</th>
<th>( z_0 ) [m]</th>
<th>( d ) [m]</th>
<th>Fetch Homogeneity</th>
<th>( z^* )</th>
<th>Data quality</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scattered crops</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Savannah, S1</td>
<td>2.3</td>
<td>5.0</td>
<td>3.5</td>
<td>0.32</td>
<td>0.44</td>
<td>1.80</td>
<td>+</td>
<td>+</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Savannah, S2</td>
<td>8.0</td>
<td>20.0</td>
<td>2.0</td>
<td>0.04</td>
<td>0.40</td>
<td>4.80</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Savannah, S3</td>
<td>9.5</td>
<td>10.0</td>
<td>2.0</td>
<td>0.19</td>
<td>0.90</td>
<td>7.10</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Savannah, S4</td>
<td>2.3</td>
<td>3.4</td>
<td>3.0</td>
<td>0.60</td>
<td>0.17</td>
<td>0.93</td>
<td>−</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Savannah, S5</td>
<td>2.5</td>
<td>6.6</td>
<td>3.0</td>
<td>0.17</td>
<td>0.25</td>
<td>1.50</td>
<td>+</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Tiger-bush, T1</td>
<td>4.0</td>
<td>40</td>
<td>20</td>
<td>0.05</td>
<td>0.44</td>
<td>2.00</td>
<td>0</td>
<td>+</td>
<td>+</td>
<td>+</td>
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<tr>
<td>Tiger-bush, T2</td>
<td>4.0</td>
<td>40</td>
<td>20</td>
<td>0.05</td>
<td>0.15</td>
<td>3.70</td>
<td>0</td>
<td>+</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>Row crops</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vineyard, R1</td>
<td>0.90</td>
<td>1.5/5.0</td>
<td>0.70</td>
<td>0.04</td>
<td>0.095</td>
<td>0.0</td>
<td>+</td>
<td>+</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>Vineyard, R2</td>
<td>0.90</td>
<td>2.5</td>
<td>0.90</td>
<td>0.13</td>
<td>0.08</td>
<td>0.31</td>
<td>+</td>
<td>+</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Cotton, R3</td>
<td>0.49</td>
<td>1.0/0.5</td>
<td>0.25</td>
<td>0.19</td>
<td>0.066</td>
<td>0.31</td>
<td>+</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Cotton, R4</td>
<td>0.38</td>
<td>1.0</td>
<td>0.30</td>
<td>0.10</td>
<td>0.16</td>
<td>0.10</td>
<td>+</td>
<td>+</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>Vineyard, R5</td>
<td>1.5</td>
<td>1.75</td>
<td>0.30</td>
<td>0.15</td>
<td>0.55</td>
<td>0.0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Vineyard, R5</td>
<td>1.5</td>
<td>1.75</td>
<td>0.30</td>
<td>0.15</td>
<td>0.20</td>
<td>0.75</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Vineyard, R6</td>
<td>2.0</td>
<td>2.0</td>
<td>1.0</td>
<td>0.50</td>
<td>0.25</td>
<td>1.40</td>
<td>−</td>
<td>+</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>Orchard, R7</td>
<td>3.7</td>
<td>7.3</td>
<td>4.0</td>
<td>0.28</td>
<td>0.23</td>
<td>0.92</td>
<td>−</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Orchard, R7</td>
<td>3.7</td>
<td>7.3</td>
<td>4.0</td>
<td>0.28</td>
<td>1.22</td>
<td>0.92</td>
<td>−</td>
<td>0</td>
<td></td>
<td>−</td>
</tr>
</tbody>
</table>

Those data sets with three plus signs in the foregoing columns, poor (−) means that at least one of the three criteria was rated poor, and moderate (0) describes the remainder.

These three criteria for data quality were all met clearly in only two cases, as indicated in Table 1. These are from measurements at the savannah sites S2 and S3. Data quality from six of the remaining experiments is compromised by being marginal with respect to at least one criterion, while data from the residual eight cases all clearly fail at least one criterion, so the \( z_0 \) and \( d \) values calculated for them are considered unreliable.

The most frequent limitation on data quality in these experiments was that the lower anemometers were placed below the level \( z^* \). Only three experiments were completely satisfactory in this respect: they were experiments S2, S3 and T1. In six cases, none of the anemometers was sited above \( z^* \) by the above criterion. In the remaining cases, some of the anemometers were below \( z^* \). For the cases R5 (b) and R7(a) only two out of five and seven levels were higher than \( z^* \) while for cases S1, S4, S5, R2, and for R3 the lowest one or two anemometers were too low. Ideally, all of these data should be discarded, but this would leave rather few data for analysis so the marginal data from S1, S5, R2, R3 and R5(b) have been retained. For the same reason, data from one site with marginal fetch, T1 have been retained. Thus, eight experimental values of \( z_0 \) and \( d \) have been deemed reliable enough to investigate the models for \( z_0 \) and \( d \) to be described in the next section. These are from the experiments S1, S2, S3, S5, T1, R2, R3 and R5(b)).

These eight results have been used along with values from R92 to test the models for \( z_0 \) and \( d \) over a wide range of \( \lambda \). Many of the results collated in R92 are unsuited for testing the R92 model because independent data on \( b \) and \( D \) are not available. The extra data include those from 5 wind-tunnel experiments with sparse arrays of cylinders, (Raupach et al., 1980), and eleven experiments on denser canopies outdoors (as given in Raupach et al., 1991). In some cases, \( z_0 \) data only were reported. As a result, there were 24 datapoints to check the R94 performance for predicting \( z_0 \) (8 new, 5 wind-tunnel and 11 outdoors from R92) and 16 datapoints to test the prediction of \( d \) with R94 (8 new, 0 wind-tunnel and 8 out-
doors from R92). To check the performance of the R92 parameterization to predict \( z_0 \), 13 datapoints were used (8 new, 5 wind-tunnel), whereas only the eight new datapoints were available for \( d \).

**MODEL FORMULATIONS**

**Raupach’s model for \( z_0 \) and \( d \).**

A brief outline of the relevant parts of R92 and R94, with the corrections given in R95 taken into account, is given here. For more detail the reader is referred to the original works.

The model for \( z_0 \) and \( d \) is based on an analysis of the deviation of the wind profile from its ideal logarithmic form within the roughness sublayer. For the wind profile in that region, R92 writes:

\[
\ln \left( \frac{h - d}{z_0} \right) = \frac{k u_*}{u_*} + \Psi_h. 
\]

(5)

where \( \Psi_h(\cdot) \) is a vegetation influence function and \( u_* \) is wind speed at the top of the canopy (m s\(^{-1}\)). To obtain values for \( z_0 \) from this equation, R92 derives expressions for \( \Psi_h \), \( d \) and \( \gamma = u_h/u_* \).

For \( \Psi_h \), R92 first notes that, experimentally, the wind profile near canopy top is linear, so he writes

\[
\frac{k(u(z) - u(z_0))}{u_*} = \frac{z - z_w}{z_w - d}. \tag{6}
\]

where \( z_w(\cdot) \) is the height at which the eddy diffusivities and, therefore, the gradient of this profile, matches that of the unmodified log profile. This \( z_w \) is lower than the height of the vegetation sublayer, \( z_* \), with which \( z_w \) is incorrectly identified in R92. R92 then argues that \( z_w - d \) will be proportional to the vertical length scale for canopy turbulence which will, in turn, be proportional to \( (h - d) \). Therefore

\[
z_w - d = c_w(h - d) \tag{7}
\]

This leads to the relationship

\[
\Psi_h = \ln(c_w) - 1 + c_w^{-1}, \tag{8}
\]

where \( c_w \) is a dimensionless constant, to be found from empirical data. Identifying \( L_\eta \) with \( (h - d) \) rather than with \( z_0 \), as in the earlier discussion, is a matter of preference at this stage. The advantage of making \( L_\eta \) proportional to \( (h - d) \) is that it allows R92 to reduce the expression for \( \Psi_h \) to the constant value given by Eq. (8).

To obtain an expression for \( d \), R92 assumed that \( d \) is the mean level of momentum absorption by a rough surface or the centroid of the drag force profile (Thom, 1971; Jackson 1981). R92 proposed that the position of this centroid is governed by the vertical spread of the strong shear layer formed behind a typical roughness element and, in particular, by the vertical distance over which the shear layer can spread before it reaches the next element downwind. This implies that \( (h - d_R)/D = u_*/u_h = \gamma^{-1} \), which leads to

\[
(h - d_R)/h = c_d(b/(h\lambda))^{1/\gamma} \gamma^{-1}, \tag{9}
\]

where \( d_R \) is the centroid of the drag force \( \tau_R \) (per unit area) acting on the roughness elements only and \( c_d \) is a constant. Accounting for the drag on the ground, \( \tau_S \), the overall centroid of the drag force profile is

\[
d = \tau_R d_R + \tau_S d_S. \tag{10}
\]

If the centroid \( d_S \) of the drag on the substrate surface is zero (no understorey, as assumed in R92 and R94), the following formula can be applied for calculation of \( d/h \):

\[
\frac{d}{h} = \frac{\beta_\lambda}{1 + \beta_\lambda} \left( 1 - c_d \left( \frac{b}{h\lambda} \right)^{1/\gamma} \right), \tag{11}
\]

where \( \beta = C_R/C_S \). Here, \( C_S \) is the drag coefficient for the substrate surface free of obstructions (when \( \lambda = 0 \)) and \( C_R \) is the drag coefficient of an isolated, surface-mounted roughness element.

A simpler description of \( d/h \) is given in R94:

\[
1 - \frac{d}{h} = \frac{1 - \exp\left(-\sqrt{c_d\Lambda}\right)}{\sqrt{c_d\Lambda}}, \tag{12}
\]

where \( \Lambda \) is the canopy area index given by \( \Lambda = 2\lambda \) and \( c_d \) a free parameter.

Eq. (11) includes the unknown quantity \( \gamma \), which also appears in the modified log profile equation (Eq. (5)). The expression for this quantity \( \gamma \) is based on the drag partitioning theory developed in R92.

In R92 and R94, sparse vegetation is idealized as a collection of upright cylinders, of height \( h \), breadth \( b \) and spacing \( D \) standing on a substrate plane, as shown in Fig. 1.

![Fig. 1 Lay-out of Raupach’s drag partition model showing isolated, cylinder-shaped, roughness elements with height, h, breadth, b and spacing, D.](image-url)
According to R92, the total drag can be divided into drag acting on the scattered elements, \( \tau_{\ell} \) (kg m\(^{-1}\) s\(^{-2}\)), and drag acting on the substrate, \( \tau_{s} \) (kg m\(^{-1}\) s\(^{-2}\)). R92 argued that the shear stress acting on the substrate is given by:

\[
\tau_{s}(\lambda) = \rho C_{S} u_{*}^{2} \exp \left[ -c_{1} \frac{u_{b}}{u_{*}} \lambda \right].
\]  

The equation for the stress on the isolated roughness elements is:

\[
\tau_{s}(\lambda) = \lambda \rho C_{S} u_{b}^{2} \exp \left[ -c_{1} \frac{u_{b}}{u_{*}} \lambda \right],
\]

where \( \rho \) is the density of air (kg m\(^{-3}\)) and the other variables are as described before. The coefficient \( c_{1} \) is an empirical coefficient determined by the rate at which an element wake spreads in the cross-stream direction. The factor \( u_{b}/u_{*} \) accounts for the sheltering of the surface and the roughness elements.

The total drag on the obstructions plus the substrate is \( \tau = \rho u_{*}^{2} \) and summing Eq. (13) and Eq. (14) gives

\[
\gamma = \frac{u_{b}}{u_{*}} = (C_{S} + \lambda C_{R})^{-\gamma} \exp(c_{1} \lambda \gamma / 2).
\]

as in R92. This is an implicit equation for \( \gamma \) which can be solved to give \( \gamma \) as a function of \( \lambda \). In R94 this formula is approximated by the explicit equation:

\[
\gamma = \frac{u_{b}}{u_{*}} = (C_{S} + \lambda C_{R})^{-\gamma},
\]

which applies strictly only in the limit as \( \lambda \to 0 \); in practice it can be used for \( \lambda \ll 0.1 \). An expression for the roughness length is then obtained by substituting either Eq. (15a) or Eq. (15b) into Eq. (5).

From Eq. (15a), it follows that \( \gamma^{-1} \) increases with \( \lambda \) up to a value of \( \lambda_{\text{max}} \) beyond which it decreases again. According to R94, the value \( \lambda_{\text{max}} \) can be interpreted as the onset of oversheltering, the point at which adding further roughness elements to the surface does not affect the bulk drag because additional elements merely shelter one another.

### Constants for the drag partition model

The equations above contain a number of coefficients whose values must be deduced from the underlying theory or found empirically by fitting the model to empirical data. These constants are \( c_{1}, c_{\psi}, C_{S}, C_{R}, c_{d}, \) and \( c_{d1} \). Of these, Raupach was able to deduce satisfactory values for \( c_{\psi} \) and \( C_{R} \) from the underlying model, while the others were deduced empirically by fitting the model to data on stress partitioning and, for \( c_{d} \) on \( z_{0} \) and \( d \), choosing the constants from within ranges that theory suggests are reasonable. R92 admits that his model for \( z_{0} \) and \( d \) is "more speculative" so the empirical determination of \( c_{1}, C_{R}, c_{d}, \) and \( c_{d1} \), will be continued using the data sets described earlier.

For \( c_{\psi} \), the value \( c_{\psi} = 2.0 \) has been adopted, giving a \( \Psi_{b} \) value of 0.20 as in R94 and R95. The drag coefficient for unobstructed bare soil is given in R92 as \( C_{S} = 0.003 \). This value should be suitable for row crop and tiger-bush sites where the ground is bare soil. However, it is probably too small for savannah sites where the ground is grassed beneath the shrubs. A new value for \( C_{S} \) can be calculated from the formula \( C_{S} = u_{b}^{2}/u_{*}^{2} \), using Eq. (5) with standard values for \( z_{0}/h \) and \( d/h \) taken from the literature for long grass and heather (Wieringa, 1993). This gives \( C_{S} = 0.018 \). Because savannah grass is usually rather sparse, the intermediate value \( C_{S} = 0.010 \) has been adopted for this type of vegetation.

In R92, an overstorey drag coefficient \( C_{R} = 0.3 \) is chosen for bush-like obstacles; this value is between 0.25 (vertical-axis cylinders) and 0.4 (cubes). Values in a range from 0.25 to 0.8 will be tested.

In R92, \( c_{d} = 0.6 \) is stated to be appropriate for most canopies. It is acknowledged that different \( c_{d} \) values may be necessary to describe different types of canopy (closed versus sparse or, within the sparse range, row-crops versus scattered crops). Here, \( c_{d} \) values were varied between 0.3 and 1.2; this being the range suggested in R92.

In R94, a value of \( c_{d1} = 7.5 \) was deemed most appropriate. Here, the constant \( c_{d1} \) was allowed to take values in the range 0.0 to 100.

In R92, a value of the constant \( c_{1} = 0.37 \) was selected. This is appropriate for cylinder-like obstacles with \( h/b \sim 1 \). Other values might be more appropriate with other values of \( b/h \). In particular, a larger value may be more appropriate for tiger-bush, where \( b/h \) is larger. In the least squares minimization here, values in the range \(-5.0 \) to 1.0 were tested. Raupach (1992) tested the model with \( c_{1} = 0.25, 0.37, 0.5 \) and 1.0.

### Other parameterizations

As mentioned in the introduction, other, much simpler, parameterizations have been used to calculate \( z_{0} \) and \( d \). The simplest express \( z_{0} \) and \( d \) as constant fractions of canopy height, allowing no dependence on \( \lambda \), thus

\[
z_{0} = k_{1}h, \quad d = k_{2}h,
\]

where \( k_{1} \) and \( k_{2} \) are empirical constants. Bruttschert (1982) gives the values \( k_{1} = 0.13 \) and \( k_{2} = 0.66 \). These are well established values taken from wind profile measurements over many agricultural crops and other dense canopies. An equation for \( z_{0} \) which includes a dependence on \( \lambda \) was proposed by Lettau (1969). It is

\[
z_{0} = k_{3}h \lambda, \quad \lambda < 1.
\]

where \( k_{3} \) is a theoretical constant, equal to 0.5.

The predictive skill of these equations will also be tested.
Statistical methods

In the model evaluations below, the ability of the various models to predict \( z_0 \) and \( d \) has been compared, using the coefficient of determination, \( r^2 \), as the measure of the performance of a mode; \( r^2 \) is a measure of the total variance accounted for by the model:

\[
r^2 = \frac{\sum_{i=1}^{n} (Y_{ob} - \bar{Y}_{ob})^2 - \sum_{i=1}^{n} (Y_{ob} - \bar{Y}_{cl})^2}{\sum_{i=1}^{n} (Y_{ob} - \bar{Y}_{ob})^2}
\]

(18)

where \( Y_{ob} \) is the \( i \)th observation, \( \bar{Y}_{cl} \) is the \( i \)th model calculation and \( \bar{Y}_{ob} \) is the mean of the observed \( Y \) data. The best model is the one giving a \( r^2 \) closest or equal to 1.0.

Comparisons will be made directly using the standard or recommended model coefficients and with coefficients optimized to fit the various data sets. This optimization will be performed using the criterion that \( r^2 \) should be a maximum.

A complication arises here because choosing coefficients which maximize the \( r^2 \) for one quantity, say \( z_0 \), may not maximize it when predicting another, say \( d \). In most cases coefficients are chosen to give a more accurate \( z_0 \)-prediction has been chosen because error in \( z_0 \) more strongly affects calculated momentum transfer in larger-scale models.

Once optimum coefficient values are found, to test whether the resulting model is significantly better than the others which may, or may not, also have been optimized,

Model Selection Criterion is used:

\[
MSC = \ln \left( \frac{\sum_{i=1}^{n} (Y_{ob} - \bar{Y}_{ob})^2}{\sum_{i=1}^{n} (Y_{ob} - \bar{Y}_{cl})^2} \right)^{2p} n
\]

(19)

where the symbols are as defined before and with \( p \) the number of fitted coefficients used in a model and \( n \) the number of observations (MicroMath, 1993). This criterion allows comparison of the predictive skill of models with various numbers of fitted coefficients and observations. The best model will be the one with the largest MSC.

Results and discussion

Here, experimental values of \( z_0 \) and \( d \) are compared with the values predicted by the various models. The eight acceptable experimental values of \( z_0 \) and \( d \) from Table 1 are used plus several of the values collated in R92 and used there to test and parameterize the drag partition model. Firstly, these experimental data are compared with the predictions of the various models using coefficient values taken directly from the literature. Comparisons are also made with coefficients selected so as to optimize their fit to the experimental data. The MSC criterion is used to compare the predictive ability of the models.

The performance of the models is summarized in Table 2. Its first two columns identify the model and whether the coefficients have been optimized (+) or not (-). The next column gives the aerodynamic characteristic for which the predictive power of the model has been tested, with the number of available observations in brackets. The following two or three columns give the model coefficients and their values. These are either the standard values or those found from Least Squares Minimization (LSM), predicting \( z_0 \) or \( d \) (see column 3). The final columns give the coefficient of determination \( r^2 \) for the aerodynamic property under optimization and the model selection criterion (MSC).

Preliminary testing of the R92 model showed that satisfactory roughness estimates could be obtained only for those cases where \( b/h \neq 1 \) and when \( C_s \) was kept at the values suggested in the previous section (0.003 and 0.01 for bare ground and understoreys, respectively). Therefore, only three coefficients were varied during the LSM for R92: \( c_d \), \( C_R \) and \( c_1 \). In the case of the R94 model, \( C_R \) and \( c_{cl} \) were optimized. In both cases, \( c_w = 2.0 \). For Eqs. (16) and (17), \( k_1 \), \( k_2 \) and \( k_3 \) were fitted to the data.

Performance of models without optimized coefficients

Values of \( r^2 \) were first calculated using standard values for the coefficients in each of the models. The \( r^2 \) values for \( d \) were high, ranging from 0.92 (R92 model) to 0.96 (R94 model). This means that prediction of \( d \) with the non-optimized coefficients was satisfactory over the entire \( \lambda \)-range.

This is not true for the \( z_0 \) values. In particular, the values of \( r^2 \) for \( z_0 \) were negative for Eqs. (16a) and (17). This means that these equations are quite unsuitable for predicting \( z_0 \) over the full range of canopies from very sparse to dense (taking the average of the 24 measurements would give a better prediction with \( r^2 = 0.0 \)). The main cause of the low \( r^2 \) value of 0.43 for the R92 model is the value of \( \Psi_h \) (~0.2) used in Eq. (5), in combination with the model coefficients originally suggested. Originally (see R92), \( \Psi_h \) had a value of 0.75 but a sign error in the equation describing \( c_w \) showed that \( c_w \sim 2.0 \) (so not 4.5) and hence \( \Psi_h \sim 0.2 \) (see R94, R95). This is also the cause of the negative \( c_1 \)-values found in Table 2 when R92 is optimized. The R94 parameterization with the original values for \( C_R \) and \( c_{cl} \) results in a \( r^2 \) for \( z_0 \) of 0.20. It must be concluded that all parameterizations with their original parameter values are unable to predict reliable \( z_0 \) values for the wide variety of canopies considered here.

Performance of models with optimized coefficients

As shown above, the current model coefficients of R92, R94 and Eqs. (16) and (17) are not very satisfactory.
Table 2. Parameter combinations and their coefficients of determination $r^2$ and MSC for $z_0$ and $d$-predictions using the ‘accepted’ (fair/good quality data, $n=8$) data set and a combination of the ‘accepted’ and extra datapoints obtained from Raupach et al. (1980) and Raupach et al. (1991). The first and the second column indicate which model has been used and whether the model coefficients were optimized (+) or not (−).

<table>
<thead>
<tr>
<th>Model</th>
<th>Optimization</th>
<th>Variable</th>
<th>Parameter</th>
<th>Coefficient of determination $r^2$</th>
<th>MSC</th>
</tr>
</thead>
<tbody>
<tr>
<td>R92</td>
<td>−</td>
<td>$z_0$ (n=13)</td>
<td>$c_d$ 0.6</td>
<td>$C_R$ 0.30</td>
<td>$c_1$ 0.37</td>
</tr>
<tr>
<td></td>
<td>−</td>
<td>$d$ (n=8)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+</td>
<td>$z_0$ (n=13)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+</td>
<td>$d$ (n=8)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R94</td>
<td></td>
<td>$c_{d1}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>−</td>
<td>$z_0$ (n=24)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>−</td>
<td>$d$ (n=16)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+</td>
<td>$z_0$ (n=24)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+</td>
<td>$d$ (n=16)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eq. (16) and (17)</td>
<td></td>
<td>$k_1$ 0.013</td>
<td>$k_2$ 0.67</td>
<td>$k_3$ 0.50</td>
<td>$&lt;0.01$</td>
</tr>
</tbody>
</table>

However, they can be improved by fitting the models to the data using a least-squares criterion.

For R92, optimization on the $r^2$ for $z_0$ yields the value of 0.20 for $c_d$, this is close to the lowest value of 0.3 suggested by Raupach. For $c_1$, the optimum value is negative, for reasons explained above. Optimization of the coefficients for $d$ also yields $c_d = 0.20$ and a negative $c_1$-value. The $C_R$ value of $<0.45$ is somewhat higher than $C_R = 0.3$, as given in R92.

For R94, optimization on $z_0$ gives $C_R$ close to the value adopted in R94, but $c_{d1} = 20.6$. The latter is considerably higher than that proposed in R94. Optimization of $c_{d1}$ on $d$ (note that Eq. (12) is independent on $C_R$) gives a similarly high value of 21.0. In R94 the much lower value of 7.5 was obtained by requiring Eq. (12) to match the $d$-data as given in Fig. 1b of R94. However, this graph consisted of only 8 datapoints representing relatively closed canopies. Furthermore, we noticed that a change of $+10\%$ in $c_{d1}$ changes $d/h$ only by $+1.5\%$, which makes a considerably higher $c_{d1}$ value plausible.

Fig. 2 shows the $r^2$ values for $z_0$ (Fig. 2a) and $d$ (Fig. 2b) as a function of their determining coefficient(s). Fig. 2b indicates that for any value of $c_{d1}$ greater than 5 a value of $r^2$ of at least 0.9 can be obtained. The optimum value of $c_{d1}$ is around 20, but Fig. 2b shows that prediction of $d$ is very insensitive to the exact value, as also found by Raupach (1994). However, for reliable values of $z_0$, $c_{d1}$ exhibits a much narrower band (see Fig. 2a), with best predictions for $c_{d1}$ ranging between 15 and 25. Because the optimum $c_{d1}$ values for prediction of $z_0$ and $d$ are so close together and keeping in mind that $z_0$ is the more important, $c_{d1} = 20.6$ will be used for further calculations.

The results of the much simpler equations of Brutsaert (1982) and Lettau (1969) are also shown in Table 2. The coefficient values $k_1$ (0.046) and $k_3$ (0.017) found by model optimization are much lower than the well-established values commonly used for productive (i.e. not very sparse) agricultural crops and forests. The optimized constant $k_2 = 0.82$ is somewhat higher than the usual value of 0.67. It is clear that these fitted
coefficients have limited value for prediction of $z_0$ over a whole range of canopy types and spacing. This is confirmed by the low $r^2$ values for these models (Table 2; $\hat{r}^2 = 0.62$ and $-0.42$).

Table 2 shows that fitting the model parameters largely improves $z_0$ predictions in all cases. The R92 model, with its extra coefficient, performed somewhat better ($\hat{r}^2 = 0.84$) than the R94 version ($\hat{r}^2 = 0.81$). The $r^2$ values of Eqs. (16a) and (17) improved too, but their predictive performance is still much lower than the other two models. The $r^2$ values for $d$ increased to values very close to 1.0.

The last two columns of Table 2 give the MSC values for the various unfitted and fitted models to establish their ranking as predictors of $z_0$ and $d$, taking into account the number of coefficients and observations. The best predictor of $z_0$ is R94 with fitted coefficients. For the prediction of $d$, the fitted R94 model again shows the best performance, closely followed by the fitted Eq. (16b).

In Fig. 3, this 'best' model for predicting $z_0$ and $d$ is compared to the original, non-optimized R94 model. Also shown are the data used in the analysis—the eight acceptable (+ and 0) values from the present survey and the values carried forward from the earlier studies (open circles). $\lambda$ has been chosen rather than $\lambda$ (see R94) for the x-axis because information on the leaf area index was not available in most cases. For the two surfaces where $\lambda$ was available (S5 and R2), the assumption that $\lambda = 2I$ worked very well.
The dashed line in both figures predicts the estimate with the original coefficients ($C_R = 0.3$ and $c_{d1} = 7.5$). The solid line represents drag partition theory with the R94 coefficients set to the optimal values given in Table 2, i.e. $c_{d1} = 20.6$ and $C_R = 0.35$.

It appears that the canopies presented in Table 1, in combination with the extra data enable verification of Raupach's theory down to values of log ($\lambda$) = -2.

**Conclusions**

It is shown that measured values of roughness length and displacement height of a wide range of canopies compare well with values calculated with Raupach’s drag partition model (Raupach, 1992; 1994; 1995), which makes this model a useful tool for $z_0$ and $d$-predictions. However, it appears that the original coefficients, as suggested in Raupach (1992; 1994; 1995), give sub-optimal estimates of $z_0$ and $d$ for this selection of canopies, which led to new suggestions for the model coefficients.

The low number of coefficients (two: $C_R$ and $c_{d1}$) and the relatively high coefficient of determination $r^2$ lead to the conclusion that the simplified version of the model (Raupach, 1994) is more appropriate than the original, comprehensive, model (Raupach, 1992). To get reliable estimates of $z_0$ ($r^2 = 0.81$) and $d$ ($r^2 = 0.99$) for up to 24 canopies, ranging from very sparse to dense, a $C_R$ value of 0.35 and a $c_{d1}$ value of about 20 appeared appropriate. The latter value is plausible although it is considerably higher than the originally suggested value of 7.5, which was based mainly on measured values of closed canopies.

The equation $d = 0.67h$ fits the data well ($r^2 = 0.93$). However, the simple rule of thumb ($z_0 = k_1h$, with $k_1 = 0.13$), gives poor estimates of $z_0$ ($r^2 < 0$) over the whole range of canopy densities. Optimization results in $k_1 = 0.046$ with $r^2 = 0.62$. Lettau's formula for $z_0$, which also involved canopy density (Lettau, 1969), generally performed badly ($r^2 < 0$), even after fitting.

It can be concluded that Raupach's drag partition models R92 and R94 with new, tuned coefficients perform significantly better than the alternative simpler models.

**Acknowledgements**

This work was financially supported by the EC (EPOC-0024-C(CD)) and NWO (750.650.37). Dr. M. Raupach is acknowledged for clarification of some of the model equations. The two anonymous reviewers and Dr. Chris Huntingford are thanked for their valuable comments.

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