Supplementary material for: Temporal dynamics of hydrological threshold events.

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This material provides full solutions to the raw and central moments of the first passage time statistics detailed in the above mentioned article and a figure showing the correspondence between analytically derived statistics and those obtained by numerical simulation.

1 Raw Moments

1.1 First raw moment: $T_1 [s_0, s_\xi]

\[ T_1 [s_0, s_\xi] = \frac{\alpha e^{(\alpha - \beta)s_\xi} - e^{(\alpha - \beta)s_0} \beta}{(\alpha - \beta)^2} - \frac{\beta(\alpha(s_\xi - s_0) + 1)}{(\alpha - \beta)^2} \]

1.2 Second raw moment: $T_2 [s_0, s_\xi]

\[ T_2 [s_0, s_\xi] = e^{-\beta s_\xi}(2e^{2s_\xi s_0} - 2e^{\alpha s_0 - \beta s_0} + \alpha s_\xi e^{3s_0} \beta + e^{\beta s_\xi} \beta^2 (-\alpha + \beta)(2\beta - \alpha(4\beta + \alpha - 2 + (\alpha - \beta)(s_0 - s_\xi)))(s_0 - s_\xi)) + 2e^{(\alpha - \beta)s_0 + \beta s_\xi}(\alpha - 3\beta + \alpha s_0 - \beta^2 s_0 + \alpha(\alpha - \beta)s_\xi) - 2e^{\alpha s_\xi} \alpha^2 \beta(-\alpha^2(s_0 - 2s_\xi) + \alpha(3 + \beta s_0 - \beta s_\xi) - \beta(5 + \beta s_\xi))) \div ((\alpha - \beta)^4 \lambda^2)\]

1.3 Third raw moment: $T_3 [s_0, s_\xi]

\[ T_3 [s_0, s_\xi] = (e^{-\beta(s_0 + 3s_\xi)}(6e^{s_0 + 3s_\xi} \alpha^6 - 6e^{\alpha s_0 + 2s_\xi} + 3s_\xi \alpha^3 \beta(-\alpha^2 s_\xi^2 - 2\alpha(1 + s_0)(3 + \beta s_0)) + 2s_\xi) + 2s_\xi(s_0 + 3s_\xi)(\alpha - \beta)) + 6e^{\alpha s_0 + \beta s_\xi}(\alpha - \beta)(\alpha + \beta)(s_0 + 3s_\xi) + 2s_\xi(s_0 + 3s_\xi)(\alpha - \beta)s_\xi - \alpha^3(s_0 - 2s_\xi)(\alpha - \beta)(s_0 + 3s_\xi) + 3s_\xi) + 6e^{\alpha s_0 + s_\xi} + 3s_\xi \alpha^2 \beta(2s_\xi + \alpha(3 + \beta s_0 - \beta s_\xi) - \beta(7 + \beta(s_0 + s_\xi))) + 3e^{\beta s_0 + \alpha s_\xi + 2s_\xi \alpha^2 \beta^2(s_0 - 2s_\xi) - 2\alpha^3(s_0 -
4. Fourth raw moment: $T_4 [s_0, s_\xi]$

$$T_4 [s_0, s_\xi] = \left( e^{\beta s_0 - 3s_\xi} - 24e^{\beta s_0 + 5s_\xi + 3s_\xi} - 24e^{\alpha s_0 + 4s_\xi} \right) + 24e^{\beta s_0 + 5s_\xi + 3s_\xi} \alpha^8$$

$$\frac{2s_\xi(3 + \beta s_0 - \beta s_\xi) + 2\alpha \beta(13 + \beta s_\xi(5 + \beta s_\xi) - \beta s_0(6 + \beta s_\xi)) + \beta^2(30 + \beta s_\xi(10 + \beta s_\xi)) + \alpha^2(6 + \beta(18s_0 + \beta s_0^2 - s_\xi(32 + 3\beta s_\xi))) (\alpha - \beta)^3}{\lambda^4}$$

2. Correspondence between analytical and numerical solutions
Figure 1: Correspondence between analytical (line) and numerical (points) of the variance of the saturation excess (a) event magnitude; and (b) inter-event time (IET); as a function of mean storm depth. Parameters used in the simulations include: $w_0 = 10$ mm, $\bar{t}_b = 3$ d, and $E_m = 2$ mm d$^{-1}$. 

(a) 

(b)