A hybrid model of self organizing maps and least square support vector machine for river flow forecasting

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Received: 18 May 2010 – Published in Hydrol. Earth Syst. Sci. Discuss.: 18 October 2010
Revised: 31 August 2012 – Accepted: 11 September 2012 – Published: 26 November 2012

Abstract. Successful river flow forecasting is a major goal and an essential procedure that is necessary in water resource planning and management. There are many forecasting techniques used for river flow forecasting. This study proposed a hybrid model based on a combination of two methods: Self Organizing Map (SOM) and Least Squares Support Vector Machine (LSSVM) model, referred to as the SOM-LSSVM model for river flow forecasting. The hybrid model uses the SOM algorithm to cluster the entire dataset into several disjointed clusters, where the monthly river flows data with similar input pattern are grouped together from a high dimensional input space onto a low dimensional output layer. By doing this, the data with similar input patterns will be mapped to neighbouring neurons in the SOM’s output layer. After the dataset has been decomposed into several disjointed clusters, an individual LSSVM is applied to forecast the river flow. The feasibility of this proposed model is evaluated with respect to the actual river flow data from the Bernam River located in Selangor, Malaysia. The performance of the SOM-LSSVM was compared with other single models such as ARIMA, ANN and LSSVM. The performance of these models was then evaluated using various performance indicators. The experimental results show that the SOM-LSSVM model outperforms the other models and performs better than ANN, LSSVM as well as ARIMA for river flow forecasting. It also indicates that the proposed model can forecast more precisely, and provides a promising alternative technique for river flow forecasting.

1 Introduction

Hydrological data such as flows and rainfall are the basic information used in the design of water resource systems. Knowledge about the characteristics and volume of river flow is very important, especially for predicting the future river flow in the monsoon season where the heavy rainfall may cause heavy river flow, potentially causing problems such as flooding and erosion. Reduced river flow is likely to restrict the supply of water for domestic use and industrial and hydroelectric power generation. Therefore, the ability to forecast future river flow would be beneficial in the field of water management and help in the design of flood protection works in urban areas and for agricultural land.

In hydrology, different types of models are used such as lumped conceptual models, physically-based models, also known as knowledge-driven modelling, and empirical models, also known as data-driven modelling. By using knowledge driven modelling, the other catchment variables such as catchment characteristics (size, shape, slope and storage characteristics of the catchment), and geomorphologic characteristics of a catchment (topography, land use patterns, vegetation and soil types that affect the infiltration) must be considered because it is hypothesized that forecasts could be improved if catchment characteristic variables which affect flow were to be included (Jain and Kumar, 2007; Dibike and Solomatine, 2001).

River flow forecast can be generated using two types of mathematical models: rainfall-runoff models and river flow models. The rainfall-runoff models use both climatic and hydrologic data, while river flow models only use the
hydrologic data. The river flow process in a catchment area is a complex process that can be affected by many interrelated physical factors. The factors affecting the river flow response of a catchment subjected to rainfall input include storm characteristics, such as intensity and duration of rainfall events, and so on. Moreover, the conceptual rainfall-runoff models need a large amount of data for calibration and validation purposes and are usually computationally expensive and very time consuming (Grayson et al., 1992; Jain and Kumar, 2007). Although combining other variables may improve their prediction accuracy, but in practice, especially in developing countries like Malaysia, such information is often either not available or difficult to obtain. Moreover, the influence of these variables and many of their combinations in generating river flow is an extremely complex physical process, especially due to the data collection of multiple inputs and parameters, which vary in space and time and which are not clearly understood (Zhang and Govindaraju, 2000; Jain and Kumar, 2007).

Owing to the complexity of this process, many researchers are focusing on river flow forecasting, which only considers the past river flow data, because it offers a rapid development and minimum information requirement (Adamowski and Sun, 2010; Kisi, 2004, 2008; Wang et al., 2009). This data-driven modelling by using historical data which are based on extracting and re-using the information that is implicitly contained in the hydrological data without taking directly into account any physical load that underlies the rainfall-runoff process provides accurate and rapid development time with minimum data information.

River flow forecasting is an important yet difficult task in the field of hydrology because predicting future events involves a decision-making process where the ability to predict future river flow will provide the right edge and assist the engineers in terms of flood control management, and provide some benefits in the areas of water supply management (Viessman et al., 1989). Accurate continuous data collections on the catchment area are needed to produce a good river flow forecast. There are many forecasting techniques that have been proposed in the literature for river flow forecasting. The most comprehensive of all popular and widely-known statistical methods used in time series forecasting is the Autoregressive Integrated Moving Average (ARIMA) model, also known as the Box Jenkins model. Several studies have shown that ARIMA can be trusted as a reliable model in water resources time series analysis (Muhamad and Hassan, 2005; Huang et al., 2004; Modarres, 2007; Fernandez and Vega, 2009; Wang et al., 2009).

Among the non-linear approaches, the Artificial Neural Network (ANN) is the most widely used for time series forecasting and has been successfully employed in the modelling of a wide range of hydrologic contexts (Maier and Dandy, 2000; Dibike and Solomatine, 2001; Bowden et al., 2005; Dolling and Varas, 2003; Muhamad and Hassan, 2005; Kisi, 2008; Wang et al., 2009; Keskin and Taylan, 2009; Luk et al., 2000; Hung et al., 2009; Affandi and Watanabe, 2007; Birkinshaw et al., 2008; Corzo et al., 2009). ANN provides an attractive alternative tool for forecasting and has shown its nonlinear modelling capabilities in data time series forecasting. However, the selection of an optimal network structure (layers and nodes) and training algorithms always needs the attention of modellers (Maier and Dandy, 2000). The network structure is usually determined by using a trial-and-error approach (Kisi, 2004).

Recently, the support vector machine (SVM) method, which was suggested by Vapnik (1995), has been used in hydrological modelling such as stream flow forecasting (Asefa, et al., 2006), flood stage forecasting (Yu et al., 2006), rainfall runoff modelling (Dibike et al., 2001; Elshorbagy et al., 2010a, b), etc. However, the standard SVM is based on the structural risk minimal principal and involves complicated quadratic programming methods, which are often time consuming and have a higher computational burden because of the required constrained optimization programming.

As a simplification of SVM, Suykens et al. (2002) proposed the use of the least squares support vector machines (LSSVM). LSSVM has been used successfully in various areas of pattern recognition and regression problems (Hambay, 2009; Kang et al., 2008). LSSVM encompasses similar advantages to SVM, but its additional advantage is that it only requires the solving of a set of linear equations, which is much easier and computationally simpler. The method uses equality constraints instead of inequality constraints and adopts the least squares linear system as its loss function, which is computationally attractive. LSSVM also has good convergence and high precision. Hence, this method is easier to use than quadratic programming solvers in the SVM method. Extensive empirical studies (Wang and Hu, 2005) have shown that LSSVM is comparable to SVM in terms of general performance. In the area of water resources, the LSSVM method has received very little attention in the literature and there are only a few applications of LSSVM in the modelling of environmental and ecological systems such as water quality prediction (Yunrong and Liangzhong, 2009).

Clustering analysis, which is the subject of active research in several fields such as statistics, pattern recognition, machine learning, and data mining, is to partition a given set of data or objects into clusters or groups or classes. It also has been applied in a large variety of applications, for example character recognition, document retrieval, etc. The goal of clustering analysis is to group similar objects together. Cluster analysis is a standard method of statistical multivariate analysis, and it can reduce large and complex datasets to a small number of data groups where members of the group share similar characteristics (Lin et al., 2006). The clustering algorithms attempt to partition data into clusters or natural groups such that the data within a cluster are as similar as possible, and data belonging to different clusters are as dissimilar as possible. Therefore, SOM pursues a goal that is conceptually different from that of clustering (Wu and Chow,
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2004). However, SOM has been successfully utilized as a first step in clustering algorithms.

The Self Organizing Map (SOM), proposed by Kohonen (2001), is one category of ANN that was first used as an information-processing tool in the fields of speech and image recognition. SOM has been successfully applied in clustering, classification, estimation, prediction and data mining (Vesanto and Alhoniemi, 2000; Kohonen, 2001). SOM can be used as a clustering tool since it converts the non-linear statistical relationship between high dimensional data into simple geometric relationships of their image points on a low-dimensional display. From that, the data points which show similar properties are placed close to each other within the output of the SOM algorithm (Budayan et al., 2009). After the SOM training is finished, one can figure out the number of clusters directly by eye according to the two-dimensional map (Lin and Chen, 2005). Clustering the dataset may seem unnecessary; however, it is an important task especially when dividing a complex problem into several smaller and simpler problems can more easily be solved compared with the original problem (Jacobs et al., 1991; Lin and Chen, 2006; Lin and Wu, 2007; Wu and Chau, 2009; Wu et al., 2009).

SOM has attracted increasing interest for water resources application, such as the classification of satellite imagery data and rainfall estimation (Murao et al., 1993), rainfall-runoff modelling (Hsu et al., 2002), typhoon-rainfall forecasting (Lin and Wu, 2009), river flood forecasting (Chang et al., 2007), water resource problems (Kalteh et al., 2008), and model evaluation (Herbst and Casper, 2008; Herbst et al., 2009). The advantages of SOM compared with the other clustering methods have been extensively discussed in the literature (Chen et al., 1995; Mangiameli et al., 1996; Lin and Chen, 2006). Mangiameli et al. (1996) showed that the SOM performed the best when compared to seven other hierarchical clustering methods. Lin and Chen (2006) recommend SOM as an alternative to the identification of homogeneous regions for regional frequency analysis where the results showed that the SOM determines the cluster membership more accurately than the K-means method and Ward’s method. In addition, the SOM is more robust than the traditional clustering methods. Lin and Chen (2005) apply the SOM clustering for predicting the groundwater head at Hsiu-Lin Station, Taiwan. Among 240 datasets, the first 192 samples are used for training and the remaining 48 samples are used for testing. The entire datasets are then mapped into $14 \times 14$ map sizes. The map is then divided into 15 regions or clusters to continue with the prediction model using RBFN.

Improving forecasting accuracy, especially in time series forecasting, is an important yet often difficult task facing decision-makers in many areas. Using hybrid models has become common practice to improve forecasting accuracy. There are several studies that show that hybrid models can be an effective way to improve the accuracy of forecasting, compared to using single models separately (Zhang, 2003; Jain and Kumar, 2007; Chen and Wang, 2007; Pai and Lin, 2005; Hsu et al., 2009). For instance, Lin and Wu (2009) proposed a combination of SOM and MLP in order to forecast the typhoon rainfall at Tanshui River Basin. SOM was used to analyze and divide the input data into distinct clusters. The second step involved an individual relationship between the input and output data constructed by a specific MLP. For evaluating the forecasting performance of the proposed model, an application was conducted. The results show that the proposed model can forecast more precisely than the model developed by the conventional neural network approach. Srinivas et al. (2009) combined a SOM and fuzzy clustering for regional flood frequency analysis for watersheds data from Indiana, USA. Results show that the proposed approach performs better in estimating flood quantiles at ungauged sites.

In recent years more hybrid models have been proposed, which combine a clustering technique with local forecasting models that are more accurate since these models are more specialised and have successfully solved many predictions problems, such as a combination of SOM with ANN (Pal et al., 2003; Lin and Wu, 2009; Wang and Yan, 2004), SOM with SVM (Cao, 2003; Fan and Chen, 2006; Fan et al., 2007; Huang and Tsai, 2009), SOM with Radial Basis Function (Lin and Chen, 2005), ANN with K-means (Corzo and Solomatine, 2007) and other models (Chang and Liao, 2006; Chang et al., 2007, 2008). Although the idea of these hybrid models is interesting and promising, it still need to be tested using a river flow time series.

Based on the same idea by Tay and Cao (2001) and Hsu et al. (2009), this study aims to explore the application of hybrid technique and to test the capability and effectiveness of the idea of hybrid modelling which combines the SOM with the LSSVM (SOM-LSSVM). The hybrid model SOM-LSSVM is then proposed for river flow forecasting in order to improve the accuracy of prediction. With the advantages of the data analysis technique developed by SOM and the capability of LSSVM, the proposed hybrid model is expected to be useful for river flow forecasting. The results of the predictions by the SOM-LSSVM model are compared with a forecasting model developed by conventional ARIMA, ANN and LSSVM models. To verify the application of this approach, the monthly river flow for Bernam River located in Selangor, Malaysia was analyzed as a case study in this research.

2 Data-driven modelling

Data-driven modelling is considered as a tool in building a model that will replace the knowledge-driven modelling in describing a physical behaviour (Solomatine et al., 2008). However, the data-driven models may not correctly represent the input–output mapping if the modelled system changed during the time when the data are collected. Therefore, it is necessary for the entire dataset to go through a statistical
analysis test to prove whether there are some other trends in the dataset. Trend analysis on time series data has been proven to be a useful tool for effective water resources planning, design, and management (Douglas et al., 2000; Hamilton et al., 2001), since trend detection of hydrological variables such as river flow provide useful information on the possibility of change tendency of the variables in the future (Yue and Wang, 2004).

Another issue that still remains a question is the number of the appropriate data that can be used to predict future river flows by using data-driven modelling. Number of data also plays an important role in predicting future river flows. For example, Kisi and Cimen (2010) used the monthly river flow data of Canakdere River and Goksudere River in their research. The observed data are 40 yr (480 months) along with an observation period between 1960 and 1999 for both stations. Jain and Kumar (2007) applied their proposed model using monthly river flow data for a period of 62 yr (1911–1972) derived from the Colorado River at Lees Ferry, CO, USA, for modelling a hydrologic time series forecasting. Usually, the monthly data are used to estimate the water demand and the water supply (Srikanthan and McMahon, 2001). Although river flow forecasting models using historical river flow time series data may lack the ability to provide physical interpretation and insight into catchment processes, they are nevertheless able to provide relatively accurate flow forecasts and become more and more popular in hydrological modelling due to their rapid development times and minimum information requirements.

3 Forecasting models

This section presents the ARIMA, ANN, LSSVM and SOM-LSSVM models used for river flow forecasting. The choice of these models in this study was due to the fact that these methods have been widely and successfully used in time series forecasting.

3.1 The Autoregressive Integrated Moving Average Model

The Box-Jenkins model, also known as Autoregressive Integrated Moving Average (ARIMA), was introduced by Box and Jenkins (1970) and has been one of the most popular approaches in the area of forecasting. The order of an ARIMA model is represented by ARIMA \((p, d, q)\) and the order of the seasonal ARIMA or SARIMA model is represented by \(\text{ARIMA} \ (p, d, q) \times (P, D, Q)_s\), where the term \((p, d, q)\) is the order of the non-seasonal and \((P, D, Q)_s\) is the order of the seasonal. The general ARIMA models are a compound of a seasonal and non-seasonal part and are represented in the following manner:

\[
\phi(B)\Phi(B^s)(1-B)^d(1-B^s)^D x_t = \theta(B)\Theta(B^s)a_t, \tag{1}
\]

where \(\phi(B)\) and \(\theta(B)\) are polynomials of order \(p\) and \(q\), respectively; \(\Phi(B^s)\) and \(\Theta(B^s)\) are polynomials in \(B^s\) of degrees \(P\) and \(Q\), respectively; \(p\) is the order of non-seasonal auto regression; \(d\) is the number of regular differencing; \(q\) is the order of the non-seasonal moving average; \(P\) is the order of seasonal autoregression; \(D\) is the number of seasonal differencing; \(Q\) is the order of seasonal moving average; \(B\) is the backward shift operator, and \(s\) is the length of the season. Random errors \(a_t\) are assumed to be independently and identically distributed with a mean of zero and a constant variance of \(\sigma^2\).

The ARIMA model involves four steps, which are the identification step, estimation step, diagnostic checking step and forecasting step. In the identification step, the sample autocorrelation function (ACF) and sample partial autocorrelation function (PACF) are used to determine whether or not the series is stationary and if it is seasonal or non-seasonal. If the series is not stationary, then a transformation called differencing is needed until the series reaches the stationary level. ACF as well as PACF are used to choose a tentative model. Once a tentative model is identified, the parameters of the model are estimated. Diagnostic checking using the ACF and PACF of the residuals is carried out, basically to check whether the model’s assumptions about the error \(a_t\) are satisfied. If the model is not adequate, a new tentative model should be identified followed by the steps of parameter estimation and model verification. The process is repeated several times until finally a satisfactory model is selected. The forecasting model is then used to compute the fitted values and forecasts values.

3.2 Artificial neural network

The ANN is flexible computing, which has been extensively studied and used for time series forecasting in many areas of science and engineering since early 1990. The most commonly used ANN in the field of water resources and hydrology is the feed forward multi layer perceptron (MLP), which consists of three layers: the first is the input layer where the data are introduced to the network, the second layer is the hidden layer where the data are processed, and the final layer is the output layer where the results of the given input are produced. The structure of a feed-forward ANN is shown in Fig. 1.

Mathematically, a three-layer MLP with \(p\) input nodes, \(q\) hidden nodes and one output node can be expressed as

\[
y_t = g \left( \sum_{j=1}^{q} w_{j} \varphi \left( \sum_{i=1}^{p} w_{i} x_{t-i} \right) \right), \tag{2}
\]

where \(y_t\) is the output layer, \(x_{t-i}\) is the input of the network, \(w_i\) is the connection weights between the input and hidden layer nodes, \(w_i\) is the connection weights between hidden and output layer nodes, and \(\varphi(.)\) and \(f(.)\) are activation functions. The most common \(\varphi(.)\) as the sigmoid function and
which is non-linearly mapped from the input space \( x \) where \( y = f(x) \) is the output vector, introducing \( \mathbf{LSSVM} \) and its use in time series forecasting. A single output as the target value. In the following, we briefly trained using a set of time series historic values as inputs and classification as well as regression. The \( \mathbf{LSSVM} \) predictor is used for the optimal control of non-linear Karush-Kuhn-Tucker systems for \( \mathbf{LSSVM} \) (Suykens, 2002). The basic \( \mathbf{LSSVM} \) is used for the optimization problem of \( \mathbf{LSSVM} \). A modification of the standard Support \( \mathbf{V} \) ector Machine \( \mathbf{SVM} \) (Suykens, 2002). The basic \( \mathbf{LSSVM} \) is used for the optimal control of non-linear Karush-Kuhn-Tucker systems for classification as well as regression. The \( \mathbf{LSSVM} \) predictor is trained using a set of time series historic values as inputs and a single output as the target value. In the following, we briefly introduce \( \mathbf{LSSVM} \) and its use in time series forecasting.

Consider a set of data \( D=\{(x_1, y_1), (x_2, y_2),\ldots,(x_n, y_n)\} \), \( x_i \in \mathbb{R}^p, y_i \in \mathbb{R}, x \) is the input vector, \( y \) is the expected output and \( n \) is the number of data. The \( \mathbf{LSSVM} \) approximate the function in the following form:

\[
y(x) = w^T \phi(x) + b,
\]

where \( \phi(x) \) represents the high dimensional feature spaces, which is non-linearly mapped from the input space \( x \). By combining the functional complexity and fitting error, the optimization problem of \( \mathbf{LSSVM} \) is given as

\[
J(w, \xi) = \frac{1}{2} w^T w + \frac{\gamma}{2} \sum_{i=1}^{n} \xi_i^2
\]

subject to:

\[
y(x) = w^T \phi(x_i) + b + \xi_i = 1, 2, 3, \ldots, n.
\]

This formulation consists of equality instead of inequality constraints. To solve this optimization problem, the Lagrange function is constructed as

\[
L(w, b, \xi; \alpha) = J(w, b, \xi) - \sum_{i=1}^{n} \alpha_i [w^T \phi(x_i) + b - y_i + \xi_i],
\]

where \( \alpha_i \) are the Lagrange multipliers, which can be positive or negative. The solution of Eq. (8) can be obtained by partially differentiating with respect to \( w, b, \xi_i \) and \( \alpha_i \).

\[
\begin{align*}
\frac{\partial L}{\partial w} &= 0 \rightarrow w = \sum_{i=1}^{n} \alpha_i \phi(x_i) \\
\frac{\partial L}{\partial b} &= 0 \rightarrow \sum_{i=1}^{n} \alpha_i = 0 \\
\frac{\partial L}{\partial \xi_i} &= 0 \rightarrow \alpha_i = y_i \\
\frac{\partial L}{\partial \alpha_i} &= 0 \rightarrow w^T \phi(x_i) + b - y_i + \xi_i = 0
\end{align*}
\]

After elimination of the variables \( w \) and \( \xi_i \), one obtains the following matrix solution:

\[
\begin{bmatrix}
1 & \phi(x)^T \\
1 & \phi(x)^T + \gamma^{-1} I
\end{bmatrix}
\begin{bmatrix}
b \\
\alpha_i
\end{bmatrix}
= \begin{bmatrix}
y \\
0
\end{bmatrix},
\]

with \( y = [y_1, y_2, \ldots, y_n] \), \( 1^T_v = [1, 1, \ldots, 1] \), \( \alpha = [\alpha_1, \alpha_2, \ldots, \alpha_l] \). The kernel function can be defined as

\[
K(x_i, x_j) = \phi(x_i)^T \phi(x_j), \quad i, j = 1, 2, \ldots, n.
\]

This finally leads to the following \( \mathbf{LSSVM} \) model for regression:

\[
y(x) = \sum_{i=1}^{n} \alpha_i K(x_i, x_j) + b,
\]

where \( \alpha_i, b \) are the solutions to the linear system and \( K(x_i, x_j) \) is a kernel function. The most popular kernel function is the Radial Basis Function (RBF), as shown in Eq. (13) (Liu and Wang, 2008; Gencoglu and Ulyar, 2009).

\[
K(x_i, x_j) = \exp \left(-\frac{\|x - x_j\|^2}{\delta^2}\right).
\]
3.4 Self organizing map

SOM, also known as the Self Organizing Feature Map (SOFM), was proposed by Professor Teuvo Kohonen and therefore sometimes called the Kohonen Map (Kohonen, 2001), is an unsupervised and competitive learning algorithm. SOM has been used widely for data analysis in some areas such as economics, physics, chemistry as well as medical applications.

The objectives of SOM are to maximize the degree of similarity of patterns within a cluster, minimize the similarity of patterns belonging to different clusters, and then present the results in a lower-dimensional space. Basically, SOM consists of two layers of artificial neurons: the input layer, which accepts the external input signals; and the output layer, also called the output map, which is usually arranged in a two-dimensional structure. Every input neuron is connected to every output neuron, and each connection has a weighting value attached to it. The architecture of SOM is shown in Fig. 2.

Output neurons will self organize to an ordered map, and neurons with similar weights are placed together. They are connected to adjacent neurons by a neighbourhood relation, dictating the topology of the map (Moreno et al., 2006). The concept of the learning algorithm for SOM is unsupervised and competitive. The training process of SOM is described below:

For simplicity, we assume that the input vector $X$ of SOM is:

$$X = [x_1, x_2, ..., x_n],$$

where $n$ is the dimension of the input vector. The weight vector connecting the input vector to the hidden neuron $i$ is denoted by

$$W_i = [w_{i1}, w_{i2}, ..., w_{in}] \quad i = 1, 2, ..., m.$$  \hfill (15)

The weights are initialised as small random numbers at the start of the training process. In competitive learning networks, the neurons compete among themselves to determine the winner by calculating the distance between the input vector and the weight vectors of all the neurons in the hidden layer. The winner $I$ is defined as the one whose weight vector is the closest to the input vector $X$:

$$I(X) = \min_{\forall i} \|X - W_i\| \quad i = 1, 2, ..., m.$$  \hfill (16)

The Euclidean distance is often used as the similarity measure for SOM. The output neuron whose weight vector has the smallest distance from the input vector is called the winning neuron.

After determining the winning neuron, the lateral intersections between the winning neuron and its neighbourhood are calculated using the topological neighbourhood function. The neighbourhood function takes the form of a radial basis function that is appropriate for representing the biological lateral interaction (Kohonen, 2001; Rui Xu, 2009):

$$h_{ji}(t) = \eta(t) \exp \left( \frac{-||r_j - r_i||^2}{2\sigma^2(t)} \right),$$  \hfill (17)

where $||r_j - r_i||$ represents the Euclidean distance between the winning neuron $i$ and the neighbouring neuron $j$, and $\sigma(t)$ is the bandwidth of the radial basis kernel function.

Next, the weights of this winning neuron are adjusted according to the input patterns using the algorithm

$$W_i(t + 1) = W_i(t) + h_{ji}(t)(X - W_i(t)), \quad t = 1, 2, ..., T.$$  \hfill (18)

where $\eta(t)$ is the learning rate at time $t$ and $W_i(t + 1)$ is the adjusted weight vector at time $t + 1$.

After the weights have been updated, the winning neuron and the neighbourhood neurons become more similar to the corresponding input pattern. The process continues until convergence has been reached. Finally, the trained SOM is obtained.

3.5 Integrating the SOM-LSSVM model

Time series is a chronological sequence of observations of data points recorded sequentially in time. Time series forecasting is used to predict future values based on past values and other variables. However, the datasets are full with non-linearity. To address these issues, this study employs a hybrid model to better predict the future river flow. In this study, a hybrid model was implemented which combines the SOM clustering algorithm with the LSSVM model, as illustrated in Fig. 3. In the first stage, the datasets are divided into several groups or clusters. In order to do this, SOM is used to cluster the whole input space into regions where
data points with similar statistical distribution are grouped together. Each group or cluster contains similar objects (Huang and Tsai, 2009). After the clustering of the data into several groups, LSSVMs are constructed for each cluster. LSSVM can conduct a better forecast for each group or cluster. As demonstrated by Tay and Cao (2001) and Hsu et al. (2009), this hybrid model can capture better results in the prediction of future river flow.

4 The study area and data

In this research, we examined the data obtained from the monthly river flow of the Bernam River located in Selangor, Peninsula Malaysia. Bernam River is located between the states of Perak and Selangor, demarcating the border between the two states. The upper Bernam River basin has been identified as the ultimate and largest source of water supply from the Bernam River, especially for irrigation and the supply of drinking water. The study area is about 1090 km² with a mean elevation of 19 m, and the Bernam River monitoring station is the downstream outlet. The location of the Bernam River catchment is shown in Fig. 4.

The monthly river flow data of the Bernam River, consisting of 516 monthly records (January 1966 to December 2008), are used in this study. The data were first tested using Mann-Kendall test in order to detect any other trends in river flow data. After that, the dataset was then split up into two parts: training and testing, where the first dataset consisting of 456 monthly records (January 1966 to December 2003) was used for training, while the final dataset containing 60 mean monthly river flows (January 2004 to December 2008) was used for testing. Training data were used exclusively for model development and testing data were used to measure the performance of the model on untrained data. The testing set was also used to evaluate the forecasting ability of the model and to compare the proposed model with others. The recorded time series data for the Bernam River are shown in Fig. 5.

Solomatine et al. (2008) suggested that when splitting data into training and testing datasets, these sets should have similar distributions of low and high flow or similar properties of the input and output variables. However, it has been found that to generalise the training and testing sets with similar properties is not an easy task. Most studies suggest that the ratio of splitting for training and testing should be [70:30, 80:20, or 90:10]. The selection of the ratio could be based on the particular problem under consideration (Zhang et al., 1998; Firat, 2007; Kisi, 2008; Wang et al., 2009). Before the training process begins, data normalisation is often performed. The river flow was normalised in the range [0.1, 0.9] by the following equation:

\[ y_t = 0.1 + \frac{x_t}{1.2(x_{\text{max}})} \]

where \( y_t \) represents the normalised data, while \( x_t \) is the actual observation value and \( x_{\text{max}} \) represents the maximum value among the actual observation values.

5 Input determination

As with any data-driven model such as ANN and LSSVM, the selection of appropriate model inputs plays an extremely important role in their successful implementation since it provides the basic information about the system being modelled. In time series forecasting, usually insufficient attention
Table 1. Model of input data.

<table>
<thead>
<tr>
<th>Model</th>
<th>Input Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>$y_t = f(x_{t-1}, x_{t-2})$</td>
</tr>
<tr>
<td>M2</td>
<td>$y_t = f(x_{t-1}, x_{t-2}, x_{t-3}, x_{t-4})$</td>
</tr>
<tr>
<td>M3</td>
<td>$y_t = f(x_{t-1}, x_{t-2}, x_{t-3}, x_{t-4}, x_{t-5}, x_{t-6})$</td>
</tr>
<tr>
<td>M4</td>
<td>$y_t = f(x_{t-1}, x_{t-2}, x_{t-3}, x_{t-4}, x_{t-5}, x_{t-6}, x_{t-7}, x_{t-8})$</td>
</tr>
<tr>
<td>M5</td>
<td>$y_t = f(x_{t-1}, x_{t-2}, x_{t-3}, x_{t-4}, x_{t-5}, x_{t-6}, x_{t-7}, x_{t-8}, x_{t-9}, x_{t-10})$</td>
</tr>
<tr>
<td>M6</td>
<td>$y_t = f(x_{t-1}, x_{t-2}, x_{t-3}, x_{t-4}, x_{t-5}, x_{t-6}, x_{t-7}, x_{t-8}, x_{t-9}, x_{t-10}, x_{t-11}, x_{t-12})$</td>
</tr>
<tr>
<td>M7</td>
<td>$y_t = f(x_{t-1}, x_{t-2}, x_{t-3}, x_{t-4}, x_{t-5}, x_{t-6}, x_{t-7}, x_{t-10}, x_{t-12})$ (Stepwise)</td>
</tr>
<tr>
<td>M8</td>
<td>$y_t = f(x_{t-1}, x_{t-2}, x_{t-12}, x_{t-13}, x_{t-14}, x_{t-24}, x_{t-25}, x_{t-26}, a_{t-24})$ (ARIMA)</td>
</tr>
</tbody>
</table>

Table 2. The result for the training and testing using ANN model.

<table>
<thead>
<tr>
<th>Data</th>
<th>Hidden Neurons</th>
<th>Training</th>
<th>Testing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>MAE</td>
<td>RMSE</td>
</tr>
<tr>
<td></td>
<td></td>
<td>MAE</td>
<td>RMSE</td>
</tr>
<tr>
<td>M1 (I =2)</td>
<td>I/2</td>
<td>0.0967</td>
<td>0.1259</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0961</td>
<td>0.1257</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0969</td>
<td>0.1257</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0971</td>
<td>0.1263</td>
</tr>
<tr>
<td>M2 (I =4)</td>
<td>I/2</td>
<td>0.1150</td>
<td>0.1506</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.1135</td>
<td>0.1500</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.1135</td>
<td>0.1489</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.1126</td>
<td>0.1482</td>
</tr>
<tr>
<td>M3 (I =6)</td>
<td>I/2</td>
<td>0.1098</td>
<td>0.1411</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0940</td>
<td>0.1258</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0909</td>
<td>0.1197</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0936</td>
<td>0.1232</td>
</tr>
<tr>
<td>M4 (I =8)</td>
<td>I/2</td>
<td>0.1125</td>
<td>0.1473</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.1100</td>
<td>0.1449</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.1075</td>
<td>0.1404</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.1067</td>
<td>0.1417</td>
</tr>
<tr>
<td>M5 (I =10)</td>
<td>I/2</td>
<td>0.1245</td>
<td>0.1602</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.1025</td>
<td>0.1359</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.1059</td>
<td>0.1402</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.1056</td>
<td>0.1396</td>
</tr>
<tr>
<td>M6 (I =12)</td>
<td>I/2</td>
<td>0.0868</td>
<td>0.1143</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0870</td>
<td>0.1154</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0828</td>
<td>0.1105</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0838</td>
<td>0.1107</td>
</tr>
<tr>
<td>M7 (I =7)</td>
<td>I/2</td>
<td>0.0863</td>
<td>0.1150</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0879</td>
<td>0.1155</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0852</td>
<td>0.1134</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0850</td>
<td>0.1127</td>
</tr>
<tr>
<td>M8 (I =10)</td>
<td>I/2</td>
<td>0.0567</td>
<td>0.0749</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0595</td>
<td>0.0756</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0553</td>
<td>0.0716</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0653</td>
<td>0.0834</td>
</tr>
</tbody>
</table>

The bold formatted line represent the best result for ANN technique.
is given to the task of selecting appropriate model inputs. Many papers reviewed failed to describe the input determination methodology used, and consequently raised doubts about the optimality of the output obtained (Bowden et al., 2005). Most researchers design experiments to help select the model inputs, while others adopt some empirical ideas. For example, Patil (1992) proposed model inputs based on 12 inputs for monthly data and four for quarterly data heuristically. Cheung et al. (1996) suggested maximum entropy principles to identify the time series lag structure. Tang and Fishwick claimed that the number of model inputs is simply the number of autoregressive (AR) moving average components in the Box-Jenkins models. Refenes et al. (2003) suggested a stepwise method for determining the input for ANN models. Roadnight et al. (1997) used cross correlation analysis or principal component analysis as a guide for determining the input. Aqil et al. (2006) employed two statistical methods, i.e. autocorrelation (ACF) and partial autocorrelation (PACF), to identify the appropriate input variables. Behzad et al. (2009) selected the best model inputs by trial and error according to minimum test errors in the ANN and SVM modelling. Corzo et al. (2009) used correlation and average mutual information analysis involving different sub-basin values using precipitation and river flow to determine the best input variables. Khashei and Bijari (2010) proposed an ARIMA model to determine the input variables in order to yield a more accurate forecasting model than ANN. The empirical results from three well-known real datasets showed that the proposed input variables can be an effective way to improve the forecasting accuracy achieved by ANN. The use of input variables from the data values of previous time series and the optimum number of input variables determined by trial and error has been reported by Firat (2007, 2008), Firat and Gungor (2007), Sivapragasam and Liong (2005), Juhos et al. (2008), Partal and Kisi (2007), among others.

Three approaches were used in this study to determine the models of input data. The first six approaches for the input data were chosen based on past river flow. The appropriate lags were chosen using a trial-and-error approach ($x_{t-1}, x_{t-2}, \ldots, x_{t-p}$, where $p$ is 2, 4, \ldots, 12). It gives the number of inputs ($I$) as 2, 4, 6, 8, 10, 12. The second and third approaches set the input vector nodes equal to the number of lagged variables from two statistical methods (i.e. stepwise multiple regression

![Graph of ACF and PACF](image)

**Fig. 6.** ACF and PACF for monthly river flow of Bernam River.

![Graph of Autocorrelation Function](image)

**ACF of Residuals for Sg Bernam**

(with 5% significance limits for the autoregressions)

![Graph of Partial Autocorrelation Function](image)

**PACF of Residuals for Sg Bernam**

(with 5% significance limits for the partial autoregressions)

**Fig. 7.** The ACF And PACF of residuals for monthly river flow of Bernam River for ARIMA (2, 0, 0) x (2, 0, 2)12 model.
Table 3. The result for the training and testing using LSSVM model.

<table>
<thead>
<tr>
<th>Data</th>
<th>Training MAE</th>
<th>RMSE</th>
<th>R</th>
<th>Testing MAE</th>
<th>RMSE</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>0.0955</td>
<td>0.1248</td>
<td>0.5494</td>
<td>0.0858</td>
<td>0.1080</td>
<td>0.5191</td>
</tr>
<tr>
<td>M2</td>
<td>0.0850</td>
<td>0.1120</td>
<td>0.6757</td>
<td>0.0860</td>
<td>0.1084</td>
<td>0.5207</td>
</tr>
<tr>
<td>M3</td>
<td>0.0829</td>
<td>0.1120</td>
<td>0.6647</td>
<td>0.0797</td>
<td>0.1000</td>
<td>0.6092</td>
</tr>
<tr>
<td>M4</td>
<td>0.0853</td>
<td>0.1134</td>
<td>0.6564</td>
<td>0.0812</td>
<td>0.1021</td>
<td>0.6037</td>
</tr>
<tr>
<td>M5</td>
<td>0.0773</td>
<td>0.1035</td>
<td>0.7767</td>
<td>0.0800</td>
<td>0.1084</td>
<td>0.5232</td>
</tr>
<tr>
<td>M6</td>
<td>0.0744</td>
<td>0.1018</td>
<td>0.7308</td>
<td>0.0744</td>
<td>0.0995</td>
<td>0.6191</td>
</tr>
<tr>
<td>M7</td>
<td>0.0720</td>
<td>0.0970</td>
<td>0.7598</td>
<td>0.0771</td>
<td>0.1019</td>
<td>0.6756</td>
</tr>
<tr>
<td>M8</td>
<td>0.0486</td>
<td>0.0633</td>
<td>0.9259</td>
<td>0.0457</td>
<td>0.0611</td>
<td>0.8769</td>
</tr>
</tbody>
</table>

analysis and the ARIMA model). Stepwise multiple regression analysis led to the selection of 7 input attributes \((x_{t-1}, x_{t-2}, x_{t-4}, x_{t-5}, x_{t-7}, x_{t-10}, x_{t-12})\).

In ARIMA, the future value of a variable is assumed to be a linear function of several past observations and random errors (Zhang, 2003; Khashei and Bijari, 2010). In this study, ARIMA \((2, 0, 0) \times (2, 0, 2)\) is selected as the best model, as described in Sect. 6.1. Therefore, the functional form of the model input data using ARIMA is

\[
y_t = f(x_{t-1}, x_{t-2}, x_{t-12}, x_{t-13}, x_{t-14}, x_{t-24}, x_{t-25}, x_{t-26}, a_{t-12}, a_{t-24}) \tag{20}
\]

where \(y_t\) is the future value, \(x_t\) is the past value at time \(t\) and \(a_t\) is the residual at time \(t\), where ARIMA is used in order to generate the residuals.

6 Evaluation of performance

There are different types of performance evaluation that have been documented in the literature (Luchetta and Manetti, 2003; Goswami et al., 2005). The performance evaluation for each model should have at least a measure of absolute error, such as mean absolute error (MAE) or root mean square error (RMSE) (Legates and McCabe, 1999). Wang et al. (2006) stated that RMSE is a good performance evaluation measurement because it is very sensitive to even small errors, in which case it is better to compare the small differences in the model’s performance.

The MAE and RMSE are defined as follows:

\[
MAE = \frac{1}{n} \sum_{i=1}^{n} |y_t - \hat{y}_t| \tag{20}
\]

\[
RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (y_t - \hat{y}_t)^2} \tag{21}
\]

where \(y_t\) and \(\hat{y}_t\) are the observed/actual and the predicted at the time \(t\). The criteria to judge the best model are relatively small for MAE and RMSE in modelling and forecasting. Other than these, the correlation coefficient \((R)\) was also used as a performance measurement. \(R\) was also used to test the ability of the model to capture the complex nature of the process that was being modelled (Jain and Kumar, 2007; Lin and Wu, 2009). It is a measure of how well the future outcomes are likely to be predicted by the model, where the predicted flows correlate with the observed flows. \(R\) is defined as

\[
R = \frac{\frac{1}{n} \sum_{i=1}^{n} (y_t - \bar{y}) (\hat{y}_t - \bar{\hat{y}})}{\sqrt{\frac{1}{n} \sum_{i=1}^{n} (y_t - \bar{y})^2} \sqrt{\frac{1}{n} \sum_{i=1}^{n} (\hat{y}_t - \bar{\hat{y}})^2}} \tag{22}
\]

where \(\bar{y}\) and \(\bar{\hat{y}}\) are the mean observed and mean predicted river flow series, respectively, and \(n\) is the number of data points. The \(R\) value is used to evaluate the linear correlation between the observed and the predicted flow. Clearly, an \(R\) value close to unity indicates a satisfactory result, while a low value or one close to zero implies an inadequate result.

7 Experiment and results

7.1 Application of the ARIMA model

Figure 5 shows the plots of the river flow time series, indicating that the time series are non-stationary and require a transformation. Samples of the autocorrelation function (ACF) and the partial autocorrelation function (PACF) for the series are plotted in Fig. 6. The ACFs curves for the monthly stream flow data decayed with mixture of sine wave pattern and exponential curves, which reflect the random periodicity of the data and indicate the need for seasonal MA terms in the model. In the PACF there were significant spikes present near lag 12 and 24, therefore indicating the series need for a seasonal AR process. The criteria used to judge the best model based on MSE show that ARIMA \((2, 0, 0) \times (2, 0, 2)\) is the best model. The model can be written as

\[
(1 - 0.352B - 0.132B^2)(1 - 0.603B^{12} - 0.395B^{24})x_t = (1 - 0.477B^{12} - 0.460B^{24})a_t, \tag{23}
\]

and can be rewritten as

\[
x_t = 0.352x_{t-1} + 0.132x_{t-2} + 0.603x_{t-12} - 0.212x_{t-13} + 0.079x_{t-14} + 0.395x_{t-24} - 0.136x_{t-25} - 0.052x_{t-26} - 0.477a_{t-12} - 0.460a_{t-24} + a_t. \tag{24}
\]

The ACF and PACF plots of the residuals of ARIMA \((2, 0, 0) \times (2, 0, 2)\) for the river flow series are shown in Fig. 7. From the residual plot of the best ARIMA model, it was observed that the selected ARIMA \((2, 0, 0) \times (2, 0, 2)\) model passed the diagnostic checks and they were all white noise. For further analysis, we decided that the ARIMA model \((2, 0, 0) \times (2, 0, 2)\) is the best to use for comparison with the others.
Table 4. The result for the training and testing using a hybrid model of SOM-LSSVM for different map sizes.

| Map Sizes | Data | Training | | | Testing | | |
|-----------|------|----------|----------|----------|----------|----------|
|           |      | MAE      | RMSE     | R        | MAE      | RMSE     | R        |
| 2 x 2     | M1   | 0.0680   | 0.0903   | 0.7964   | 0.0740   | 0.0872   | 0.7508   |
|           | M2   | 0.0655   | 0.0860   | 0.8205   | 0.0767   | 0.0963   | 0.6574   |
|           | M3   | 0.0758   | 0.1031   | 0.7259   | 0.0785   | 0.1020   | 0.6072   |
|           | M4   | 0.0686   | 0.0931   | 0.7925   | 0.0752   | 0.0975   | 0.6456   |
|           | M5   | 0.0770   | 0.1045   | 0.7250   | 0.0784   | 0.0988   | 0.6322   |
|           | M6   | 0.0869   | 0.1135   | 0.6495   | 0.0794   | 0.1022   | 0.5931   |
|           | M7   | 0.0758   | 0.1051   | 0.7126   | 0.0764   | 0.1011   | 0.6082   |
| 3 x 3     | M1   | 0.0747   | 0.0997   | 0.7445   | 0.0640   | 0.0860   | 0.7376   |
|           | M2   | 0.0532   | 0.0681   | 0.8908   | 0.0683   | 0.0879   | 0.7254   |
|           | M3   | 0.0760   | 0.1029   | 0.7271   | 0.0736   | 0.0917   | 0.7099   |
|           | M4   | 0.0736   | 0.0995   | 0.7517   | 0.0733   | 0.0908   | 0.7019   |
|           | M5   | 0.0721   | 0.1008   | 0.7504   | 0.0734   | 0.0951   | 0.6650   |
|           | M6   | 0.0685   | 0.0914   | 0.8049   | 0.0794   | 0.1067   | 0.5421   |
|           | M7   | 0.0735   | 0.0974   | 0.7599   | 0.0703   | 0.0971   | 0.6474   |
| 4 x 4     | M1   | 0.0747   | 0.0997   | 0.7445   | 0.0640   | 0.0860   | 0.7376   |
|           | M2   | 0.0532   | 0.0681   | 0.8908   | 0.0683   | 0.0879   | 0.7254   |
|           | M3   | 0.0760   | 0.1029   | 0.7271   | 0.0736   | 0.0917   | 0.7099   |
|           | M4   | 0.0736   | 0.0995   | 0.7517   | 0.0733   | 0.0908   | 0.7019   |
|           | M5   | 0.0721   | 0.1008   | 0.7504   | 0.0734   | 0.0951   | 0.6650   |
|           | M6   | 0.0685   | 0.0914   | 0.8049   | 0.0794   | 0.1067   | 0.5421   |
|           | M7   | 0.0735   | 0.0974   | 0.7599   | 0.0703   | 0.0971   | 0.6474   |
| 5 x 5     | M1   | 0.0747   | 0.0997   | 0.7445   | 0.0640   | 0.0860   | 0.7376   |
|           | M2   | 0.0532   | 0.0681   | 0.8908   | 0.0683   | 0.0879   | 0.7254   |
|           | M3   | 0.0760   | 0.1029   | 0.7271   | 0.0736   | 0.0917   | 0.7099   |
|           | M4   | 0.0736   | 0.0995   | 0.7517   | 0.0733   | 0.0908   | 0.7019   |
|           | M5   | 0.0721   | 0.1008   | 0.7504   | 0.0734   | 0.0951   | 0.6650   |
|           | M6   | 0.0685   | 0.0914   | 0.8049   | 0.0794   | 0.1067   | 0.5421   |
|           | M7   | 0.0735   | 0.0974   | 0.7599   | 0.0703   | 0.0971   | 0.6474   |

Table 5. Comparative performance between ARIMA, ANN, LSSVM and SOM-LSSVM during the testing period.

<table>
<thead>
<tr>
<th>Model</th>
<th>MAE</th>
<th>RMSE</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARIMA</td>
<td>0.0767</td>
<td>0.1042</td>
<td>0.5842</td>
</tr>
<tr>
<td>ANN</td>
<td>0.0605</td>
<td>0.0837</td>
<td>0.8610</td>
</tr>
<tr>
<td>LSSVM</td>
<td>0.0457</td>
<td>0.0611</td>
<td>0.8769</td>
</tr>
<tr>
<td>SOM-LSSVM</td>
<td>0.0370</td>
<td>0.0492</td>
<td>0.9222</td>
</tr>
</tbody>
</table>

7.2 Application of ANN model

In this study, a typical three layer ANN model with a log-sigmoid transfer function from the input layer to the hidden layer, and a linear function from the hidden layer to an output layer, are used for forecasting monthly river flow time series. The input and target data were normalised in the range [0.1, 0.9], because a sigmoid function was employed as the transfer function. The network was trained for 5000 epochs using the conjugate gradient descent backpropagation algorithm with a learning rate of 0.001 and a momentum coefficient of 0.9. The eight models of input data (M1–M8) with various numbers of input structures are trained and tested by ANN models, and the optimal number of neuron in the hidden layer was identified using several guidelines.

To help avoid the problem of over-fitting, some researchers have provided empirical rules to restrict the number of hidden nodes. In order to select an appropriate architecture, the following guidelines were used: “I/2” proposed by...
Kang (1991), “I” proposed by Tang and Fishwick (1993), “2I” proposed by Wong (1991), and “2I + 1” proposed by Hecht-Nielsen (1990), where \( I \) is the number of inputs.

It is common to use one test set for both validation and testing purposes, particularly with small datasets (Zhang, 2003). For ANN experiment, the dataset was split into two sections: training set and test set where the test set was used for validation and testing purposes, as demonstrated by Zhang (2003) and Kisi (2004). The networks that yielded the best results with the lowest MAE and RMSE and largest \( R \) from the testing set were selected as the best ANN for the corresponding series. The effects of changing the number of hidden nodes on the RMSE, MAE and \( R \) are shown in Table 2.

Table 2 shows the performance of ANN varying with the number of nodes in the hidden layer. For the training and testing period, the M8 model with 20 hidden nodes obtained the best results for MAE, RMSE and \( R \), with statistics of 0.0553, 0.0716 and 0.9163, respectively. While in the testing phase, the M8 model with 20 hidden neurons was the best ANN, RMSE and \( R \) with statistics of 0.0606, 0.0837 and 0.8610, respectively. Hence, the ANN (10, 20, 1) has been selected as the most appropriate ANN model for the Bernam River.

7.3 Application of the LSSVM model

There is no single proven theory that can be used to guide the selection of the number of inputs. In this study, the same input structures of the datasets M1 to M8 were used. The RBF was used as the kernel function for this study. In order to better evaluate the performance of the proposed approach, we considered a grid search of \((\gamma, \sigma^2)\) within the range 10 to 1000, and \(\sigma^2\) in the range 0.01 to 1.0. For each hyper parameter pair \((\gamma, \sigma^2)\) in the search space, 5-fold cross validation on the training set was performed to predict the prediction error. Table 3 shows the performance results obtained in the training and testing period of the LSSVM approach.

By considering these training and testing periods, the lowest MAE and RMSE and the largest \( R \) for the series of data were calculated from the M8 model, resulting in statistics of 0.0486, 0.0633, 0.9259 and 0.0457, 0.0611, 0.8769, respectively.

7.4 Application of the hybrid SOM-LSSVM model

Determining the appropriate map sizes of clustering is very important for cluster validity and efficiency. For a SOM of large map sizes, input patterns will be grouped into a large number of clusters, which would cause each neuron to memorise one of the input patterns, although some clusters may only have one or two members. Such clustering results are not suitable for the forecasting analysis. On the other hand, if the map size is too small, then many different data groups might be lumped into the same category and the SOM will fail to show the topological relationships of the input patterns. Since there is no systematic or standard method for finding the optimal number of map sizes in the clustering algorithms, the optimal map size is obtained depending on the requirements of the user. In this paper, four map sizes are utilized, Kohonen of \(2 \times 2, 3 \times 3, 4 \times 4\) and \(5 \times 5\). When SOM is applied to perform cluster analysis, a SOM of a small dimension is the first choice. If the clustering result is reasonable and satisfactory, then the cluster analysis is accepted. Otherwise, a SOM of a larger dimension is chosen to analyse the input patterns, and this continues until a satisfactory result is obtained. In this study, only 4 clusters were considered to investigate the impacts of the number of map sizes on the performance. The same parameters were used as for the LSSVM’s parameters for the single LSSVM model. Table 4 shows the predicted values of SOM-LSSVM for the various numbers of map sizes.

7.5 Comparison

In this section, the predictive capabilities of the proposed SOM-LSSVM model are compared with ARIMA, ANN and LSSVM for the Bernam monthly river flow. Furthermore, the MAE, RMSE, and \( R \) are used to evaluate the performance of the ARIMA, ANN, LSSVM and SOM-LSSVM models. The statistical results of the different models are summarised in Table 5. From Table 5, it can be noted that the SOM-LSSVM model has the best performance with the lowest MAE and RMSE, and the largest \( R \) for the testing phase. The single LSSVM is the second best model, followed by ANN. As can be seen in Tables 5, ARIMA has the worst performance based on MAE, RMSE and \( R \).

In the testing phase, the SOM-LSSVM model improved the ARIMA model with about a 52.78% and 51.76% reduction in RMSE and MAE values, respectively, and with a 57.86% improvement of the forecast results for the \( R \) value. SOM-LSSVM also produced some improvement over the ANN model with about a 45.16% and 38.84% reduction in RMSE and MAE, respectively, and some improvement in the forecast value of about 7.1% for \( R \). As with LSSVM, the SOM-LSSVM model resulted in some improvement over LSSVM as well, with about a 19.47% reduction in RMSE, 19.03% reduction in MAE and improvement of 5.16% in the \( R \) value.

Figure 8 shows the results obtained from the four models ARIMA, ANN, LSSVM and SOM-LSSVM compared with the actual river flow data for the last sixty months for the testing phase on the Bernam River data. From Fig. 8, all the models gave a close approximation to the actual observation data. It indicates that ARIMA, ANN, LSSVM and SOM-LSSVM fit the monthly mean river flow better, and that these models are applicable for river flow forecasting. Meanwhile, Fig. 9 shows the scatter plots for the Bernam River, indicating that the estimates of the LSSVM and SOM-LSSVM models are closer to the actual river flow data than those of ARIMA and ANN. However, the value of \( R \) and the fit line...
Fig. 8. Predicted and observed river flow during testing period by ARIMA, ANN, LSSVM and SOM-LSSVM for Bernam River.

Fig. 9. The scatter plot of predicted and observed river flow during testing period using ARIMA, ANN, LSSVM and SOM-LSSVM for Bernam River.
equation coefficients of the SOM-LSSVM are superior compared to the other models. The results indicate that the best performance can be obtained by the SOM-LSSVM model, followed by the LSSVM, ANN and ARIMA models. SOM-LSSVM can also give a better prediction performance than ARIMA, ANN and LSSVM time series approaches. The results obtained in this study indicate that the SOM-LSSVM model is a powerful tool, as well as an alternative method for modelling river flow time series.

8 Conclusions

To improve the performance of river flow forecasting, a hybrid model based on a combination of SOM and LSSVM was proposed to predict monthly river flows. Before applying these models, the selections of the input data variables were conducted to determine the capability and suitability of the models to predict river flows. By using an evaluation on the performance test, the input data variables based on the ARIMA model were chosen as the optimal input factors. Next, SOM clustering technique was used to analyze these input data variables. The SOM algorithm clustered the entire input data into several disjointed clusters and after decomposing the data, LSSVM was used to predict the river flow. The result shows that the performance of river flow forecasting can be significantly enhanced by using the proposed hybrid SOM-LSSVM model.

To illustrate the capability of the SOM-LSSVM model, monthly river flow data from Bernam River were analyze in this study. The river flow data were varied per the number of input data used in the experiments. The number of input data variables were determine using the three approaches of past observation, stepwise regression analysis and ARIMA model. The experimental results show that SOM-LSSVM performs better than other models such as ARIMA, ANN and LSSVM. Through the comparison of four different models applied in monthly river flow forecasting, it can be concluded that SOM-LSSVM provides a promising alternative technique for river flow time series forecasting. It can also be concluded that the selections of input data variables also play an important role in the prediction as well as the number of Kohonen map sizes. In this study, only river flows data are considered for analysis, so future research can further test the idea of the hybrid model by employing the rainfall-runoff data, and using other clustering techniques such as K-Means or Fuzzy C-Means. Since this is an exploratory analysis of the hybrid SOM-LSSVM, the model should be further tested with another type of data with a variety of sample sizes to test the feasibility and ability of the SOM-LSSVM model. The idea of the hybrid model can also be tested on other time series data such as rainfall forecast, weather forecast, economic and so on to prove its capability and usability.

Acknowledgements. This research is supported by Zamalah Scholarship, Universiti Teknologi Malaysia and in part of E-Science, Ministry of Science, Technology and Innovation ( MOSTI) fundamental research grant scheme under vote number 79346.

Edited by: D. Solomatine

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