Comparison of the results by the present method and the fourth order Runge-Kutta method

For comparing the result obtained by the present method and that by the fourth order Runge-Kutta method, we take the Example 5.8 in the book of K. Subramanya (2009). The book title is “Flow In Open Channels.” The followings show: (1) the content of Example 5.8, (2) the numerical code for Matlab by using the present method (Jan & Chen’s method), (3) the numerical code for Matlab by using the standard fourth order Runge-Kutta method, and the comparison of the results by the present method and fourth order Runge-Kutta method.

Example 5.8
A river 100 m wide and 3.0 m deep has an average bed slope of 0.0005. Estimate the length of GVF profile produced by a low dam which raises the water surface just upstream of it by 1.50 m. Assume the channel cross-section is rectangular, the Manning coefficient n equals 0.035, and the normal depth of flow is 3.0 m. Find the GVF profile (M1 curve).

Numerical code for Matlab by the present method (Jan & Chen’s method) :
clear all;clc;tic
%------Basic Information-------
h0=3; % Normal depth
S0=0.0005; % Average bed slope
n=0.035; % Manning coefficient
h1=4.5; % Water depth (m) at boundary condition, x=0
g=9.81; % Gravitational acceleration (m/s²)
q=h0^((5/3)*S0^(1/2))/n; % Unit-width discharge for wide channels by Manning equation (m³/s/m)
hc=(q^2/g)^(1/3); % Critical depth (m)
N=10/3; % Hydraulic exponent for uniform-flow computation
M=3; % Hydraulic exponent for critical-flow computation in wide channels
hEND=3.03; % Upstream water depth (m)
xs=0; % Boundary condition at which the start of GVF calculation
u1=h1/h0; % Dimensionless water depth at downstream BC x=0
XS=xs*S0/h0; % Dimensionless longitudinal coordinate
ld=hc*h0; % Dimensionless critical depth
a=1; % Parameter used in Gaussian hypergeometric function
b1=−1/N; % Parameter used in Gaussian hypergeometric function
c1=b1+1; % Parameter used in Gaussian hypergeometric function
b2=(M-1)/N; % Parameter used in Gaussian hypergeometric function
c2=b2+1; % Parameter used in Gaussian hypergeometric function
z1=u1^(-N); % Dimensionless water-depth parameter used in GHF

hypergeomBF1= hypergeom([a,b1], c1, z1) % GHF-1 at BC
hypergeomBF2= hypergeom([a,b2], c2, z1) % GHF-2 at BC

%------------M1 Curve calculated by Jan &Chen’s method --------------------
for i=1:100000 % Upper-limit of calculation steps
h(i)=h1-0.01*(i-1); % Water depth (m) at a specified longitudinal location to be determined
u(i)=h(i)/h0; % Dimensionless variable of water depth
z(i)=u(i)^(-N); % Dimensionless variable used in GHF
if y(i)<hEND % Condition for the end of GVF calculation
break
end
Xx0(i)=XS+(u(i)*hypergeom([a,b1], c1, z(i))-u1*hypergeomBF1)+
ld^M*(u(i)^(-M+1)*hypergeom([a,b2], c2, z(i))-u1^(-M+1)*hypergeomBF2)/(M-1);
Xx(i)=Xx0(i)*h0/(S0); % transfer the longitudinal coordinate to be dimensional
if h(i)==hEND % Condition for the end of GVF calculation
break
end
h=h(1:length(Xx)); % Let storage length for water depth equal that for distance Xx
toc

Numerical code for Matlab by the standard fourth order Runge-Kutta method :
clear all;clc;format long;tic
%-------Basic Information-------
h0=3; % Normal depth (m)
S0=0.0005; % Average bed slope
n=0.035; % Manning coefficient
h1=4.5; % Water depth (m) boundary condition : x=0
g=9.81; % Gravitational acceleration (m/s^2)
q=h0^(5/3)*S0^(1/2)/n; % Unit-width discharge for wide channels by Manning equation (m^3/s/m)
hc=(q^2/g)^(1/3); % Critical depth (m)
B=100; % Width of a rectangular channel (m)
\( Q = q \times B; \) % Flow discharge in a rectangular channel (m\(^3\)/s)

\( T = B; \) % Top width of a rectangular channel (m)

\( dx = 1; \) % Distance interval (m)

\( L = -8644; \) % Computation end of longitudinal distance (m)

\( nn = L / dx; \) % Number of calculation steps

\( h = []; \) % Water depth (m) for the longitudinal datum

%----------Calculation by Standard Fourth Order Runge-Kutta method solved by Matlab----------

for \( i = 1:nn \)

\( LL(i) = dx \times i; \) % Storage for longitudinal distance length

%--------K1----------

\( A_1 = B \times h_1; \) % Cross-section area of flow in a rectangular channel (m\(^2\))

\( P_1 = 2 \times h_1 + B; \) % Wetted perimeter of flow in a rectangular channel (m)

\( R_1 = A_1 / P_1; \) % Hydraulic radius (m)

\( V_1 = Q / A_1; \) % Average velocity (m/s)

\( S_{f1} = (V_1^2 \times n^2 / R_1^{4/3}); \) % Energy slope evaluated by Manning equation

\( F_1 = (S_0 - S_{f1}) / (1 - Q^2 \times T / (g \times A_1^3)); \) % Function in the basic differential equation of GVF

\( K_1 = dx \times F_1; \) % First parameter (m)

%--------K2----------

\( A_2 = B \times (h_1 + K_1 / 2); \) % Cross-section area of flow in a rectangular channel (m\(^2\))

\( P_2 = 2 \times (h_1 + K_1 / 2) + B; \) % Wetted perimeter of flow in a rectangular channel (m)

\( R_2 = A_2 / P_2; \) % Hydraulic radius (m)

\( V_2 = Q / A_2; \) % Average velocity (m/s)

\( S_{f2} = (V_2^2 \times n^2 / R_2^{4/3}); \) % Energy slope evaluated by Manning equation

\( F_2 = (S_0 - S_{f2}) / (1 - Q^2 \times T / (g \times A_2^3)); \) % Function in the basic differential equation of GVF

\( K_2 = dx \times F_2; \) % Second parameter (m)

%--------K3----------

\( A_3 = B \times (h_1 + K_2 / 2); \) % Cross-section area of flow in a rectangular channel (m\(^2\))

\( P_3 = 2 \times (h_1 + K_2 / 2) + B; \) % Wetted perimeter of flow in a rectangular channel (m)

\( R_3 = A_3 / P_3; \) % Hydraulic radius (m)

\( V_3 = Q / A_3; \) % Average velocity (m/s)

\( S_{f3} = (V_3^2 \times n^2 / R_3^{4/3}); \) % Energy slope evaluated by Manning equation

\( F_3 = (S_0 - S_{f3}) / (1 - Q^2 \times T / (g \times A_3^3)); \) % Function in the basic differential equation of GVF

\( K_3 = dx \times F_3; \) % Third parameter (m)

%--------K4----------

\( A_4 = B \times (h_1 + K_3 / 2); \) % Cross-section area of flow in a rectangular channel (m\(^2\))

\( P_4 = 2 \times (h_1 + K_3 / 2) + B; \) % Wetted perimeter of flow in a rectangular channel (m)

\( R_4 = A_4 / P_4; \) % Hydraulic radius (m)

\( V_4 = Q / A_4; \) % Average velocity (m/s)
\[ S_{f4} = \left( V^4 \right)^2 \left( n^2 \right) / R^4 \left( 4/3 \right) \]; % Energy slope evaluated by Manning equation
\[ F_4 = \left( S_0 - S_{f4} \right) / \left( 1 - Q^2 * T / \left( g * A^4 \right) \right) \]; % Function in the basic differential equation of GVF
\[ K_4 = d \times F_4 \]; % Fourth parameter (m)

\[ h_2 = h_1 + \left( K_1 + 2 \times K_2 + 2 \times K_3 + K_4 \right) / 6 \]; % Water depth (m) at the subsequent step
\[ h_1 = h_2 \]; % Renew water depth (m) for next-step calculation
\[ h = [h \ h_2] \]; % Storage length for water depth

\textbf{Comparison of the results by the present method and fourth order Runge-Kutta method}

The numerical error in water depth obtained by the standard fourth order Runge-Kutta method is about 2% at the longitudinal distance x = -8 km.